11.1 Introduction

Air compressors are used for supplying high-pressure air. There are many uses of high-pressure air in the industry. The main uses of high-pressure (compressed) air are:

- to drive compressed air engines (air motors) used in coal mines,
- to inject or spray fuel into the cylinder of a Diesel engine (air injection Diesel engine),
- to operate drills, hammers, air brakes for locomotives and railway carriages, water pumps and paint sprays,
- to start large (heavy) Diesel engines,
- to clean workshop machines, generators, automobile vehicles, etc.,
- to operate blast furnaces, gas turbine plants, Bessemer convertors used in steel plants, etc.,
- to cool large buildings and air crafts, and
- to supercharge I.C. engines.

There are mainly two types of air compressors viz. reciprocating air compressors and rotary air compressors. Reciprocating air compressors are similar to reciprocating engines where a piston reciprocates inside a cylinder. In rotary air compressors, air is compressed due to rotation of impeller or blades inside a casing. Air compressors are driven by engines or electric motors. In this chapter the theory of reciprocating air compressors is discussed in details and principles of working of reciprocating compressed air motors and rotary compressors are explained in brief.

11.2 Reciprocating Air Compressors

The principal parts of a reciprocating air compressor are the same as that for an engine. The reciprocating air compressor may be single-acting (air is admitted to one side of the piston only) or double-acting (air is admitted to each side of the piston alternatively), and may be single-stage or multi-stage. In a multi-stage compressor, the air is compressed in several stages instead of compressing the air fully in a single cylinder. This is equivalent to a number of compressors arranged in series. The pressure of air is increased in each stage. Single-stage compressors are used for delivery pressures up to 10 bar, three-stage compressors for pressure up to 200 bar and two-stage compressors for pressures in between 10 to 200 bar. The average piston speed of a reciprocating air compressor is limited to about 300 to 400 metres per minute to reduce friction wear.

11.3 Single-stage Air Compressor

The sectional view of an air cooled, single-stage, single-acting reciprocating air compressor is shown in fig. 11-1. Both intake (suction) and discharge (delivery) valves
are disc type and are automatic in their action. They are opened and closed by difference in the air pressure acting on their two sides. When the pressures are equal on their two sides, they are kept closed by light springs. During the outward or suction stroke, the pressure in the cylinder falls below the atmospheric pressure as a result of which the intake valve opens and air is drawn from the atmosphere into the cylinder. During the inward or compression stroke, as a result of the piston action the pressure of the air in the cylinder gradually increases and reaches a value sufficiently above the receiver pressure. The high pressure of air, thus produced, overcomes the resistance of the spring on the discharge valve and causes the valve to open and discharge takes place from the cylinder to the receiver. The receiver is a simple vessel which acts as a storage tank. The compressor is driven by some form of prime mover (electric motor or engine). When the compressor is to be started against tank (receiver) pressure, the prime mover will have to supply very high starting torque. To avoid this, hand unloader (fig. 11-1) is used for releasing pressure from the compressor cylinder when the compressor is stopped.

11.3.1 Indicator Diagram: The events described above can be conveniently represented by p–v diagram shown in fig. 11-2. The diagram is drawn for a compressor without clearance. During the suction stroke the charge of air is drawn into the cylinder along line 4-1 at constant pressure $p_1$, which is slightly below that of the atmosphere. At point 1, the piston completes the suction (outward) stroke and starts on its return (compression) stroke. All valves being closed, the air is now compressed along the compression curve 1-2. At point 2, pressure $p_2$ is reached which is slightly higher than the pressure in the receiver. The discharge valve at this point opens and the delivery of the compressed air takes place along line 2-3 at pressure $p_2$. The piston has now reached the left hand end of the cylinder and again starts on its suction stroke and the pressure in the cylinder will be lowered again to $p_1$ and the cycle of operations will be repeated. The net work required for compression and delivery of the air per cycle is represented by the area 1-2-3-4 (fig. 11-2).

The amount of work done on the air will depend upon the nature of the compression curve. If the compression occurs very rapidly in a non-conducting cylinder so that there is no heat transfer, the compression will be practically isentropic. If it is carried out slowly so that the heat of the compression is extracted from the air by the jacket cooling water, the compression will approach isothermal. However, in actual practice
neither of these conditions can be fulfilled and the actual compression will be between isentropic and isothermal.

11.3.2 Isothermal Compression Versus Isentropic Compression: The slope of the compression curve, represented by the law \( pv^n = C \), depends upon the value of the index \( n \). A large value of \( n \) will give comparatively a steeper curve. The law for an isothermal or hyperbolic compression is \( pv = C \), where the value of index \( n \) is unity. The law for an isentropic compression is \( pv^n = C \). Since the value of \( \gamma \) for air is 1.4, the isentropic curve will be steeper than isothermal curve. Figure 11-3 shows curves representing an isentropic compression (1-2') and an isothermal compression (1-2''). The middle curve (1-2) shows curve, which is obtained in actual practice. The curve is polytropic \((pv^n = C)\) having a value of \( n \) nearly equal to 1.3 for the water cooled cylinder.

The isentropic work required to be done per cycle on the air is represented by the area 4-1-2'-3 (fig. 11-3). If the compression carried out had been isothermal, the slope of the compression curve would be less than that of isentropic and the isothermal work done would be represented by the area 4-1-2''-3 which is evidently less than the isentropic work done represented by the area 4-1-2'-3. Therefore, it follows that an isothermal compression is economical and efficient, since less work is required to carry it out, while an isentropic compression requires more amount of work to be supplied. Compression curve with values of index \( n \) between 1 and 1.4 will fall within the isothermal and isentropic curves. Thus, it will be seen that the work required for compression and delivery of air per cycle decreases as the value of \( n \) decreases.

The theoretical indicator diagram for a single-stage compressor without clearance is shown in fig. 11-4. Let \( p_1 \) in \( \text{N/m}^2 \) (newtons per square metre) or \( \text{Pa} \) (pascals) and \( v_1 \) in \( \text{m}^3 \) represent initial condition of the air before compression, and \( p_2 \) in \( \text{N/m}^2 = \text{Pa} \), the final delivery pressure after compression, Then,

(a) Work required to be done on the air \( W \), per cycle assuming compression curve to be polytropic, i.e., \( pv^n = C \), is given by area 1-2-3-4 of fig. 11-4.

Now area, 1-2-3-4 = area 0-a-2-3 plus area a-2-1-b minus area b-1-4-0

\[
= p_2v_2 + \frac{p_2v_2 - p_1v_1}{n-1} - p_1v_1
\]
\[ W = \frac{n}{n-1} (p_2v_2 - p_1v_1) \text{ Joule per cycle} \quad \ldots (11.1a) \]

or \[ W = \frac{n}{n-1} mR(T_2 - T_1) \text{ Joule per cycle} \quad \ldots (11.1b) \]

From eqn. (11.1a) taking \( p_1v_1 \) outside the bracket,

Work required, \[ W = \frac{n}{n-1} p_1v_1 \left[ \frac{p_2v_2}{p_1v_1} - 1 \right] \]

But for polytropic compression, \( p_1v_1^n = p_2v_2^n \). Hence, \( \frac{v_2}{v_1} = \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} \)

Substituting the value of \( \frac{v_2}{v_1} \) in the above equation,

Work required per cycle (or per revolution, if compressor is single-acting),

\[ W = \frac{n}{n-1} p_1v_1 \left[ \left( \frac{p_2}{p_1} \right) \times \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \]

\[ = \frac{n}{n-1} p_1v_1 \left[ \left( \frac{p_2}{p_1} \right)^n - 1 \right] \text{ Joule per cycle} \quad \ldots (11.2) \]

This equation gives the work required in Joules per cycle (or per revolution, if the compressor is single-acting) in compressing and delivering the air.

It should be noted that units of pressure and volume in eqn. (11.2) are N/m² or Pa and m³ respectively.

Indicated power of the compressor = \( \frac{W \times N}{60} \) J/sec. or W  
\( \ldots (11.3) \)

where \( W = \) work required in Joules per cycle, and

\( N = \) No. of cycles performed per minute = r.p.m. for single-acting compressor.

If \( p_1v_1 \) in eqn. (11.2) is substituted by \( mRT_1 \), then work required per cycle,

\[ W = \frac{n}{n-1} mRT_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ Joule per cycle and} \]

Work required per kg of air,

\[ W = \frac{n}{n-1} RT_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ Joule} \quad \ldots (11.4) \]
Indicated power of the compressor

\[ W = W \times \text{mass of air delivered per second J/sec. or } W \]  ... (11.5)

where \( W \) = work required in Joules per kg of air.

(b) If the compression is isentropic \((p^\gamma = C)\), the index \( n \) will be replaced by \( \gamma \) in eqn. (11.2) and eqn. (11.4), and then

Work required, \( W = \frac{\gamma}{\gamma - 1} \frac{d_1}{d_2} \left( \frac{p_2}{p_1} \right)^{\gamma - 1} \) Joule per cycle  ... (11.6a)

Work required per kg of air,

\[ W = \frac{\gamma}{\gamma - 1} R T_1 \left( \frac{p_2}{p_1} \right)^{\gamma - 1} \] Joule  ... (11.6b)

(c) If the compression is isothermal \((p v = C)\), then work required per cycle,

\[ W = \text{area 1-2-3-4 (fig. 11-4)} \]

\[ = \text{area a-2-1-b plus area 0-a-2-3 minus area b-1-4-0} \]

\[ \therefore W = \frac{d_1}{d_2} \log_e \left( \frac{V_1}{V_2} \right) + p_2 v_2 - p_1 v_1 = \frac{d_1}{d_2} \log_e \left( \frac{V_1}{V_2} \right) \text{ (since } p_2 v_2 = p_1 v_1) \]

\[ = \frac{d_1}{d_2} \log_e \left( \frac{p_2}{p_1} \right) \text{ Joule per cycle} \]  ... (11.7a)

Work required per kg of air, \( W = R T_1 \log_e \left( \frac{p_2}{p_1} \right) \) Joule  ... (11.7b)

11.3.3 Approximation of Isothermal Compression : Although isothermal compression is economical, it is not possible to achieve it in practice. To have an isothermal compression, the compressor will have to be run extremely slow, while in practice it is driven at high speed so that as much air as possible is compressed in a given time.

Since, there is saving of work by compressing air isothermally, it is necessary to make an attempt to obtain approximately an isothermal compression. Three methods are adopted to achieve this object while still running the compressor at high speed.

The three methods adopted are:

— cooling the air during compression by spraying cold water into the cylinder,
— cooling the air during compression by circulating cold water through the cylinder jacket, and
— adopting multi-stage compression with inter-stage cooling.

Cold water spray : In this method, cold water is sprayed into the cylinder during compression. The cooling, thus done, reduces the temperature of the air and the compression curve will be approximately of the form \( p v^{1/2} = C \). This means that the compression is brought nearer to isothermal which results in the saving of work.

Water jacket : In this method, the heat of compression is extracted by circulating cold water in the cylinder jacket thereby keeping the temperature rise as small as possible. This keeps the compression near to isothermal as shown in fig. 11-3.
Multi-stage compression: In this method, the compression of air is carried out in two or more stages in separate cylinders. The pressure of the air is increased in each stage. It is a common practice to provide intercoolers between the cylinders of multi-stage compressor, for the purpose of cooling the compressed air to atmospheric (intake) temperature before entering the succeeding (next) stage. It is this cooling between the cylinders that keeps the compression very near to isothermal as shown in fig. 11-6.

11.4 Two-stage Air Compressor

A two-stage air compressor with water jacketed cylinders and intercooler is shown in fig. 11-5(a). The suction in the L.P. cylinder (fig. 11-5b) ends at 1 and the air is drawn in the cylinder at pressure $p_1$. The air is then compressed polytropically to $2'$. The L.P. cylinder then discharges (delivers) the air along line $2-p_2$ to the intercooler where air is cooled at constant pressure $p_2$, to the original (intake) temperature corresponding to point 1 by the circulating cold water. When air is cooled in the intercooler to intake temperature corresponding to point 1, the cooling is perfect. The air in cooling at constant pressure suffers a reduction of volume from $2'-p_2$ to $p_2$. The cooled air is then drawn into the H.P. cylinder (fig. 11-5c) along line $p_2-2$ for the second stage compression, where it is compressed polytropically to the final pressure $p_3$ along line $2-3$, and then delivered to the receiver (not shown) at constant pressure $p_3$ along line $3-p_3$.

In fig. 11-6, the combined ideal indicator diagram is shown for the low pressure and high pressure cylinders of a single-acting, two-stage air compressor with
perfect-intercooling. The low pressure cylinder diagram is shown as $p_1-1-2'-p_2$, and high pressure cylinder diagram as $p_2-2-3-p_3$. The reduction of work required due to intercooling is shown by the shaded area 2-3'-3" (fig. 11-6). When cooling is perfect, i.e., when air is cooled to intake temperature in the intercooler ($T_1 = T_2$), the point 2 will lie on the isothermal line 1 - 3" as shown in fig. 11-6.

It may be noted that each stage will increase the pressure of air while the intake temperature $T_1$ (corresponding to point 1) is maintained same at the end. The isothermal line during the process has been approximated as shown by the diagram, and the shaded area 2-3'-3" shows the saving of work as a result of this approximated isothermal.

### 11.4.1 Imperfect-Intercooling

Figure 11-7 represents the indicator diagram of a two-stage air compression with imperfect-intercooling. Let the compression follow the law $pV^n = constant$ and the intercooling be incomplete (imperfect) so that the point 2 has not reached the isothermal line, i.e., point 2 does not lie on the isothermal curve 1-3".

Let $p_1$ in N/m² = Pa and $V_1$ in m³ represent condition of air entering low-pressure cylinder, and $p_2$ and $V_2$ represent condition of air entering high-pressure cylinder, and $p_3$ be final delivery pressure of air, then the total work done for compression and delivery of air per cycle will be the sum of the work done in each cylinder. Work done in L.P. cylinder is shown by the area $p_1-1-2'-p_2$ and in H.P. cylinder by the area $p_2-2-3-p_3$ (fig. 11-7). The saving of work done due to imperfect intercooling is shown by the shaded area 2-3'-3".

Hence, from eqn. (11.2),

Work required in L.P. cylinder per cycle = \[ \frac{n}{n-1} p_1 V_1 \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \] Joule.

Work required in H.P. cylinder per cycle = \[ \frac{n}{n-1} p_2 V_2 \left( \frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 1 \] Joule.

:. Total work required per cycle,

\[ W = \frac{n}{n-1} \left[ p_1 V_1 \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + p_2 V_2 \left( \frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 1 \] Joule \hspace{1cm} (11.8a)

Indicated power of the compressor = \[ \frac{W \times N}{60} \] J/sec. or watt.

where $N$ = No. of cycles per min. = r.p.m. for single-acting compressor, and $W$ = work required in Joules per cycle.
If \( p_1 v_1 \) and \( p_2 v_2 \) in eqn. (11.8a) are substituted by \( mRT_1 \) and \( mRT_2 \) respectively, then work done per kg of air can be written as

\[
W = \frac{n}{n-1} \left[ RT_1 \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + RT_2 \left( \frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 1 \] Joules. \quad \ldots (11.8b)

If the compression is isentropic, \( \gamma \) should be substituted for \( n \) in eqns. (11.8a) and (11.8b).

11.4.2 Perfect-Intercooling: If intercooling is perfect or complete (fig. 11-6), the point 2 will lie on the isothermal line, i.e., point 2 will coincide with point 2", then \( p_1 v_1 = p_2 v_2 \).

Substituting this in eqn. (11.8a), total work required per cycle,

\[
W = \frac{n}{n-1} p_1 v_1 \left( \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left( \frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right) \] Joules. \quad \ldots (11.9a)

Indicated power of the compressor = \( \frac{W \times N}{60} \) J/sec. or \( W \).

where, \( N = \) no. of cycles per min. = r.p.m. for single-acting compressor,
and \( W = \) work required in Joules per cycle.

If \( p_1 v_1 \) in eqn. (11.9a) is substituted by \( mRT_1 \), then work required per kg of air may be written as

\[
W = \frac{n}{n-1} RT_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left( \frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right] \] Joules. \quad \ldots (11.9b)

Indicated power of the compressor

\[ = W \times \text{mass of air delivered per second J/sec or watt.} \quad \ldots (11.9c)\]

where, \( W = \) work required in Joule per kg of air.

Referring to fig. 11-6,

Heat rejected to intercooler per min. = \( m k_p (T_{2'} - T_2) \) kJ \quad \ldots (11.10)

where, \( m = \) mass of air compressed per minute,
\( k_p = \) specific heat of air at constant pressure,
\( T_{2'} = \) temperature of air before entering the intercooler, and
\( T_2 = \) temperature of air after leaving the intercooler.

11.4.3 Ideal Intercooler Pressure: It may be noted from fig. 11-6 that saving in work increases as intercooling is increased. When intercooling is perfect, i.e., when air is cooled to intake temperature in the intercooler, point 2 lies on isothermal curve and there is maximum saving. In this case, work required is given by eqn. (11.9a). It may be further noted that this saving in work required also varies with the chosen intercooler pressure \( p_2 \). When the initial pressure \( p_1 \) and final pressure \( p_3 \) are fixed, the best value of the intercooler pressure \( p_2 \) shall be fixed to give minimum work. This value
of $p_2$ can be found by differentiating expression of $W$ (eqn. 11.9a) with respect to $p_2$ and equating it to zero.

The eqn. (11.9a) can be re-written by putting $\frac{n-1}{n} = y$.

$$W = \text{constant} \times \left[ \left( \frac{p_2}{p_1} \right)^y + \left( \frac{p_3}{p_2} \right)^y - 2 \right]$$

Differentiating and equating it to zero for minimum work,

$$\frac{dW}{dp_2} = y \left( \frac{p_2}{p_1} \right)^{y-1} \left( \frac{p_3}{p_2} \right)^y \left( \frac{p_1}{p_2} \right)^y - y \left( \frac{p_3}{p_2} \right)^y \left( \frac{p_2}{p_1} \right)^y = 0$$

Dividing throughout by $y$ and re-arranging,

$$\left( \frac{p_2}{p_1} \right)^{y-1} \left( \frac{p_3}{p_2} \right)^y = \left( \frac{p_3}{p_2} \right)^y \left( \frac{p_2}{p_1} \right)^y$$

$$\therefore \frac{p_2}{p_1} = \left( \frac{p_1 p_3}{p_2} \right)^y$$

Taking $y^{th}$ root throughout (i.e. both sides),

$$p_2^2 = p_1 p_3 \text{ or } \frac{p_2}{p_1} = \frac{p_3}{p_2} \text{ or } p_2 = \sqrt{p_1 p_3}$$

... (11.11)

This shows that for minimum work required or maximum efficiency, the intercooler pressure is the geometric mean of the initial and final pressures, or pressure ratio in each stage is the same.

This (eqn. 11.11) gives the best value of $p_2$ when $p_1$ and $p_3$ are given for minimum work or maximum efficiency.

Substituting $\frac{p_2}{p_1}$ for $\frac{p_3}{p_2}$ in eqn. (11.9a),

Min. work required per cycle, $W = 2 \frac{n}{n-1} p_1 v_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ Joule} \quad \ldots (11.12a)$

Since, $\frac{p_2}{p_1} = \frac{p_3}{p_2}$, then $\left( \frac{p_2}{p_1} \right)^2 = \frac{p_2}{p_1} \times \frac{p_3}{p_2} = \frac{p_3}{p_1}$

$$\therefore \left( \frac{p_3}{p_1} \right)^{\frac{1}{2}} = \frac{p_2}{p_1}$$

Substituting $\left( \frac{p_3}{p_1} \right)^{\frac{1}{2}}$ for $\left( \frac{p_2}{p_1} \right)$ in eqn. (11.12a),

Minimum work required per cycle (in terms of $p_1$ and $p_3$) can be written as

$$W = 2 \frac{n}{n-1} p_1 v_1 \left[ \left( \frac{p_3}{p_1} \right)^{\frac{2n}{n}} - 1 \right] \text{ Joule per cycle.} \quad \ldots (11.12b)$$

Minimum indicated power of the compressor = $W \times \frac{N}{60} \text{ J/sec. or watt.} \quad \ldots (11.12c)$
If \( p_1 v_1 \) in eqns. (11.12a) and (11.12b) is substituted by \( mRT_1 \), then minimum work required per kg of air may be written as

\[
W = 2 \frac{n}{n-1} RT_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \] Joule \hspace{1cm} \text{...(11.13a)}
\]

and

\[
W = 2 \frac{n}{n-1} RT_1 \left[ \left( \frac{p_3}{p_2} \right)^{\frac{n-1}{2n}} - 1 \right] \] Joule \hspace{1cm} \text{...(11.13b)}
\]

Minimum indicated power of the compressor

\[
= W \times \text{mass of air delivered per sec. J/sec. or } W \] \hspace{1cm} \text{...(11.13c)}
\]

Thus, conditions for maximum efficiency or minimum work required are:

1. The air is cooled to initial (intake) temperature in the intercooler, i.e., intercooling is perfect.
2. The pressure ratio in each stage is the same.
3. The work required for each stage is the same.

These conditions can also be extended for three-stage air compressors.

11.5 Three-stage Air Compressor

A three-stage air compressor with L.P. and I.P. intercoolers is shown in fig. 11-8.

In fig. 11-9, the combined indicator diagram for a three-stage air compressor is shown. The air having volume \( v_1 \) and pressure \( p_1 \) is compressed polytropically to pressure \( p_2 \) in the first or low-pressure cylinder, then delivered through L.P. intercooler to the second or intermediate pressure cylinder at pressure \( p_2 \), its volume shrinking to \( v_2 \). The air having volume \( v_2 \) and pressure \( p_2 \) is then compressed polytropically to \( p_3 \) in the I.P. cylinder, and is then delivered through I.P. intercooler to the high-pressure cylinder (third cylinder) at pressure \( p_3 \), its volume shrinking to \( v_3 \). This air having pressure \( p_3 \) and volume \( v_3 \) is then compressed polytropically to volume \( v_4 \) in the H.P. cylinder and then delivered to the receiver at pressure \( p_4 \).

As in the case of two-stage compression, the shaded area in fig. 11-9 represents the saving of work due to using three cylinders with inter-stage cooling instead of single-stage.

(a) Work required per cycle when intercooling is imperfect, i.e., air is not cooled to intake temperature in the intercoolers,

\[
W = \frac{n}{n-1} p_1 v_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} p_2 v_2 \left[ \left( \frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} p_3 v_3 \left[ \left( \frac{p_4}{p_3} \right)^{\frac{n-1}{n}} - 1 \right] \] Joule \hspace{1cm} \text{...(11.14a)}
\]

If \( p_1 v_1, p_2 v_2, \) and \( p_3 v_3 \) in eqn. (11.14a) are substituted by \( mRT_1, mRT_2 \) and \( mRT_3 \) respectively, then work required per kg of air can be written as
When intercooling is perfect, $p_1v_1 = p_2v_2 = p_3v_3$ (fig. 11-9).

Substituting $p_1v_1$ for $p_2v_2$ and $p_3v_3$ in eqn. (11.14a), work required per cycle,

$$W = \frac{n}{n-1} p_1v_1 \left[ \left( \frac{p_2}{p_1} \right)^{n-1} + \left( \frac{p_3}{p_2} \right)^{n-1} + \left( \frac{p_4}{p_3} \right)^{n-1} - 3 \right] \text{ Joule} \quad \ldots \quad (11.15a)$$

If $p_1v_1$ in eqn. (11.15a) is substituted by $mRT_1$, then work required per kg air may be written as,

$$W = \frac{n}{n-1} RT_1 \left[ \left( \frac{p_2}{p_1} \right)^{n-1} + \left( \frac{p_3}{p_2} \right)^{n-1} + \left( \frac{p_4}{p_3} \right)^{n-1} - 3 \right] \text{ Joule} \quad \ldots \quad (11.15b)$$

(c) Work required is minimum when $\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3}$
Substituting \( \frac{p_2}{p_1} \), \( \frac{p_3}{p_2} \) and \( \frac{p_4}{p_3} \) in eqn. (11.15a),

Min. work required per cycle, \( W = \frac{3n}{n-1} p_1 v_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \) Joule \quad (11.16a)

Now, for minimum work, \( \frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \left( \frac{p_4}{p_1} \right)^{\frac{1}{3n}} \)

Substituting \( \left( \frac{p_4}{p_1} \right)^{\frac{1}{3n}} \) for \( \frac{p_2}{p_1} \), \( \frac{p_3}{p_2} \) and \( \frac{p_4}{p_3} \) in eqn. (11.16a),

Min. work required per cycle, \( W = \frac{3n}{n-1} p_1 v_1 \left[ \left( \frac{p_4}{p_1} \right)^{\frac{n-1}{3n}} - 1 \right] \) Joule \quad (11.16b)

Minimum indicated power of the compressor = \( W \times \frac{N}{60} \) J/sec. or \( W \) \quad (11.16c)

where \( N = \) no. of cycles per min. = r.p.m. for single-acting compressor, and

\( W = \) work required in Joule per cycle.

If \( p_1 v_1 \) in eqns. (11.16a) and (11.16b) is substituted by \( mRT_1 \),

then minimum work required per kg of air compressed and delivered may be written as,

\[ W = \frac{3n}{n-1} RT_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \] \quad (11.17a)

\[ W = \frac{3n}{n-1} RT_1 \left[ \left( \frac{p_4}{p_1} \right)^{\frac{n-1}{3n}} - 1 \right] \) Joule \quad (11.17b)

Minimum indicated power of the compressor

\[ = W \times \text{mass of air delivered per second J/sec. or } W \] \quad (11.17c)

where \( W = \) work required in Joule per kg of air

(d) Equation (11.16b) can readily be extended to \( m \) stages.

Then \( \frac{3n}{n-1} \) will be replaced by \( \frac{mn}{n-1} \) and similarly \( \frac{n-1}{3n} \) by \( \frac{n-1}{mn} \)

while \( \frac{p_4}{p_1} \) will always refer to delivery and suction pressures, i.e. \( \frac{p_{m+1}}{p_1} \).

Then minimum work required per cycle with complete intercooling,

\[ W = \frac{mn}{n-1} p_1 v_1 \left[ \left( \frac{p_{m+1}}{p_1} \right)^{\frac{n-1}{mn}} - 1 \right] \) Joule \quad (11.18)
All the expression derived above for work required refer to the work actually done or required to be done on the air, and the power derived from these expressions will be referred to as indicated power or air power.

11.5.1 Advantages of multi-stage compression: The advantages of multi-stage compression are as follows:

- Reduction in power required to drive the compressor owing to compression being approximated to isothermal,
- Better mechanical balance of the whole unit and uniform torque,
- Increased volumetric efficiency as a result of the lower delivery pressure in the L.P. cylinder clearance space,
- Reduced leakage loss owing to reduced pressure difference on either sides of the piston and valves,
- Less difficulty in lubrication due to the lower working temperature, and
- Lighter cylinders.

11.6 Air Compressor Terminology

The following terminology should be well understood before attempting to estimate the performance of the air compressor.

**Free air delivered** is the volume of air delivered under the conditions of temperature and pressure existing at the compressor intake, i.e., volume of air delivered at surrounding air temperature and pressure. In the absence of any given free air conditions, these are generally taken as 1.01325 bar and 15°C.

**Capacity of a compressor** is the quantity of the free air actually delivered by a compressor in cubic metres per minute.

**Piston displacement** is the volume in cubic metre (m³) obtained as the product of the piston area in m² and the piston stroke in metre.

**Displacement per minute** is the product of the piston displacement and working strokes per minute. For multi-stage compressors, the displacement is based on low-pressure cylinder only, since it determines the amount of air passing through the other cylinder.

**Indicated power or air power** is the power determined from the actual indicator diagram taken during a test on the compressor. It is calculated in the same manner as is done in the case of a steam engine and internal combustion engine.

**Shaft or brake power** is the power delivered to the shaft of the compressor or the power required to drive the compressor. The compressor may be driven by an engine or an electric motor.

\[
\text{Shaft or brake power} - \text{Air or indicated power} = \text{Friction power}
\]

and Mechanical efficiency, \( \eta_m = \frac{\text{Air (indicated) power}}{\text{Shaft (brake) power}} \)

**Isothermal power** of a compressor is calculated from the theoretical indicator diagram drawn on the basis of an assumption that the compression is isothermal.

(a) Referring to eqn. (11.7a) for a single-stage compressor without clearance,

\[
\text{Isothermal work required per cycle, } W = p_1 v_1 \log_e \left( \frac{p_2}{p_1} \right) \text{ Joule} \quad \ldots (11.19a)
\]
Isothermal power = \( p_1 v_1 \log_e \left( \frac{p_2}{p_1} \right) \times \frac{N}{60} \) J/sec. or W (11.19b)

where \( N \) = no. of cycles per minute.

If \( p_1 v_1 \) in eqn. (11.19a) is substituted by \( mRT_i \), then isothermal work required per kg of air may be written as,

\[
W = RT_i \log_e \left( \frac{p_2}{p_1} \right) \text{ Joule}
\]

Isothermal power = \( W \times \text{ mass of air delivered per sec.} \) J/sec. or W (11.20b)

(b) For a two-stage compressor,

Isothermal work required per cycle, \( W = p_1 v_1 \log_e \left( \frac{p_3}{p_1} \right) \) Joule (11.21a)

Isothermal power = \( W \times \text{ mass of air delivered per sec.} \) J/sec. or W (11.21b)

If \( p_1 v_1 \) in eqn. (11.21a) is substituted by \( mRT_i \), then isothermal work required per kg of air may be written as

\[
W = RT_i \log_e \left( \frac{p_3}{p_1} \right) \text{ Joule}
\]

Isothermal power = \( W \times \text{ mass of air delivered per sec.} \) J/sec. or W (11.22b)

(c) Similarly, for a three-stage compressor,

Isothermal work required per cycle, \( W = p_1 v_1 \log_e \left( \frac{p_4}{p_1} \right) \) Joule (11.23a)

Isothermal power = \( W \times \text{ mass of air delivered per sec.} \) J/sec. or W (11.23b)

where \( N \) = no. of cycles per min.

Adiabatic power is calculated from a theoretical indicator diagram drawn on the basis of an assumption that the compression is an ideal adiabatic, i.e., isentropic.

Adiabatic work required, \( W = \frac{\gamma}{\gamma - 1} (p_2 v_2 - p_1 v_1) \) Joule per cycle

This equation for the adiabatic work required may be expressed in more convenient form by writing its equivalent,

\[
W = \frac{\gamma}{\gamma - 1} mR (T_2 - T_1) = \frac{\gamma}{\gamma - 1} mRT_i \left( \frac{T_2}{T_1} - 1 \right)
\]

Since \( \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} \),

\[
\therefore \text{ Adiabatic work required, } W = \frac{\gamma}{\gamma - 1} mRT_i \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \text{ Joule per cycle}
\]
Thus, for a single-stage air compressor,

\[
\text{Adiabatic (isentropic) power} = \frac{\gamma}{\gamma - 1} p_1 v_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \times \frac{N}{60} \text{ J/sec. or W.} (11.24b)
\]

where \( N \) = no. of cycles per min.

\[
\text{Isothermal efficiency} = \frac{\text{Isothermal power in watts}}{\text{Indicated or actual power in watts}} \quad (11.25a)
\]

\[
\left[ \frac{\text{Overall isothermal efficiency or}}{\text{Compressor efficiency}} \right] = \frac{\text{Isothermal power}}{\text{Shaft power or brake power required to drive the compressor}}
\]

\[
= \frac{p_1 v_1 \log_e \left( \frac{p_2}{p_1} \right) \times \frac{N}{60} \text{ watt}}{\text{Shaft power or brake power required in watts to drive the compressor}} \quad (11.25b)
\]

where \( N \) = no. of cycles per min.

\[
\left[ \frac{\text{Isentropic efficiency or ideal}}{\text{adiabatic efficiency}} \right] = \frac{\text{Isentropic power in watts}}{\text{Shaft power required in watts}}
\]

\[
= \frac{\gamma}{\gamma - 1} p_1 v_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \times \frac{N}{60} \text{ watt} \quad (11.26)
\]

where \( N \) = no. of cycles per min.

\[\text{Volumetric efficiency of an air compressor is the ratio of the actual volume of the free air at standard atmospheric conditions discharged in one delivery stroke, to the volume swept by the piston during the stroke. The standard atmospheric conditions (S.T.P.) is actually taken as pressure of 760 mm Hg (1.01325 bar) and temperature 15°C in this connection. Thus,}
\]

\[
\text{Volumetric efficiency} = \frac{\text{Volume of free air delivered per stroke}}{\text{Volume swept by piston per stroke}} \quad (11.27)
\]

The value of volumetric efficiency varies between 70 to 85 per cent according to the type of compressor.

The volumetric efficiency decreases as the clearance volume increases. Other factors that lower the volumetric efficiency are:

- Valve leakage, specially at the inlet valve,
- Obstruction at inlet valves,
- Piston ring leakage, which allows air to pass from one side of the cylinder to the other,
- Heating of air by contact with hot cylinder walls, and
— Very high speed of rotation.

With the decrease of volumetric efficiency, the capacity (quantity of free air delivered) of the compressor decreases.

**Problem-1:** A single-cylinder, single-acting reciprocating air compressor has a cylinder of 24 cm diameter and linear piston speed of 100 metres per minute. It takes in air at 100 kPa (100 kN/m²) and delivers at 1 MPa (1 MN/m²). Determine the indicated power of the compressor. Assume the law of compression to be \( pv^{1.25} = \text{constant} \). The temperature of air at inlet is 288 K. Neglect clearance effect.

Given \( p_1 = 100 \text{ kPa} = 100 \times 10^3 \text{ Pa} \); \( p_2 = 1 \text{ MPa} = 1,000 \text{ kPa} = 1,000 \times 10^3 \text{ Pa} \).

\[
\frac{p_2}{p_1} = \frac{1,000 \times 10^3}{100 \times 10^3} = 10
\]

Swept volume in m³/min. = \( \frac{\pi}{4} d^2 \times l \times \text{r.p.m.} \)

(where, \( l \) = piston strokes in metre, and \( d \) = diameter of the cylinder in metre)

\[
\therefore \text{Swept volume} = \frac{\pi}{4} \times \left(\frac{24}{100}\right)^2 \times \frac{100}{2} \text{ m³/min.}
\]

( \( \therefore \) piston speed = \( 2 \times l \times \text{r.p.m.} = 100 \) metres/min.)

\[
= \frac{2.261}{60} \text{ m³/sec.}
\]

Referring to fig. 11-10 and using eqn. (11.2), Work required per sec.,

\[
W = p_1 v_1 \times \frac{n}{n-1} \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] \text{ J/sec.}
\]

[where, \( p_1 \) is pressure in Parcals (Pa) and volume of air compressed, \( v_1 \) is in m³ per sec.]

\[
\therefore W = \left(100 \times 10^3\right) \times \frac{2.261}{60} \times 1.25 \left[ \left(100\right)^{1.25-1} - 1 \right]
\]

\[
= 1,886.16 \times (1.5848 - 1) = 11,030 \text{ J/sec. or } 11,030 \text{ W}
\]

\( \therefore \) Indicated power of the compressor = 11,030 W i.e. 11.03 kW

**Problem-2:** A single-acting, single-stage air compressor developing indicated power of 11 kW, runs at 200 r.p.m. and has a linear piston speed of 100 metres per min. If the suction pressure and temperature are 100 kPa and 15°C respectively and delivery pressure is 1,000 kPa, calculate the dimensions of the compressor cylinder. Assume the law of compression to be \( pv^{1.25} = \text{constant} \). Neglect clearance effects.

Referring to fig. 11-10, and considering polytropic compression 1-2, \( p_1 v_1^n = p_2 v_2^n \),
\[
\frac{v_1}{v_2} = \left( \frac{p_2}{p_1} \right)^\frac{1}{n-1} = \left( \frac{10}{1} \right)^{1.25} = 6.31
\]

Using eqn. (11.1a), work done per cycle, \( W = p_2 v_2 + \frac{p_2 v_2 - p_1 v_1}{n-1} - p_1 v_1 \)

\[
\therefore \text{ M.E.P.} = \frac{\text{Work done per cycle in kJ}, W}{\text{Displacement volume in m}^3, v_1}
\]

\[
= \frac{p_2 v_2 + \frac{p_2 v_2 - p_1 v_1}{n-1} - p_1 v_1}{v_1} = \frac{p_2 v_2}{v_1} - p_1 - \left( \frac{p_2 v_2}{v_1} - p_1 \right) \frac{1}{n-1}
\]

\[
= 1,000 \times \frac{1}{6.31} - 100 + \frac{1,000 \times 6.31}{0.25} - 100 = 292.4 \text{ kPa.}
\]

Piston stroke, \( l = \frac{\text{piston speed per min.}}{\text{piston strokes per min.}} = \frac{100}{2 \times 200} = 0.25 \text{ metre or 25 cm} \)

Indicated power of compressor = \( p_m \times l \times a \times n \text{ watt.} \)

\[
\left( \text{where } n = \text{ no. of cycles per sec. = } \frac{200}{60} \right)
\]

i.e. \( 11 \times 10^3 = 292.4 \times 10^3 \times 0.25 \times 0.7854 \times \left( \frac{d}{100} \right)^2 \times \frac{200}{60} \)

\[
\therefore d^2 = \frac{11 \times 10^3 \times 10^4 \times 60}{292.4 \times 10^3 \times 0.25 \times 0.7854 \times 200} = 574.79
\]

\[
\therefore d = \sqrt{574.79} = 23.98 \text{ cm.}
\]

Problem-3: A single-acting, single-stage air compressor is belt driven from an electric motor at 300 r.p.m. The cylinder diameter is 20 cm and the stroke is 24 cm. The air is compressed from one atmosphere to 8 atmospheres and the law of compression is \( p v^{1.25} = \text{constant.} \) Find the power of the electric motor if the transmission efficiency is 96 per cent and the mechanical efficiency of the compressor is 85 per cent. Neglect clearance effect.

Swept volume,

\[
v_1 = \frac{\pi}{4} d_1^2 \times l = \frac{\pi}{4} \left( \frac{20}{100} \right)^2 \times \left( \frac{24}{100} \right) = 0.00754 \text{ m}^3.
\]

Referring to fig. 11-11 and using eqn. (11.2), work required per cycle,

\[
W = \frac{n}{n-1} p_1 v_1 \left\{ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} \text{ Joule.}
\]

\[
= \frac{1.25}{1.25 - 1} \times (1.01325 \times 10^5) \times 0.00754 \times \left\{ \left( \frac{8}{1} \right)^{\frac{1.25 - 1}{1.25}} - 1 \right\}
\]

\[
= 5 \times (1.01325 \times 10^5) \times 0.00754 \times (1.515 - 1) = 1,967.27 \text{ Joule/cycle.}
\]
Work required per sec. = work done per cycle × r.p.s.

\[ = 1.96727 \times 5 = 9.836 \text{ J/sec.} \]

or 9.836 W

\[ \therefore \text{Indicated power} = 9.836 \text{ kW} \]

Taking into consideration the given mechanical efficiency of the compressor only,

\[ \text{Power required} = \frac{9.836}{0.85} = 11.57 \text{ kW} \]

\[ \therefore \text{Power of the electric motor, (considering the transmission efficiency also)} \]

\[ = \frac{11.57}{0.96} = 12.05 \text{ kW} \]

Problem-4 : The piston of an air compressor has displacement of 9.5 m³ per minute. If the pressure and temperature at the intake are 100 kPa and 25°C respectively, and the compressor in 21/4 minutes raises the pressure in 1.45 m³ capacity air receiver to 1,500 kPa and temperature 60°C, find the volumetric efficiency of the compressor.

Assume initial pressure and temperature in the receiver as 100 kPa and 25°C. Take \( R = 0.287 \text{ kJ/kg K} \) for air.

Mass of air initially in the receiver can be obtained by applying characteristic equation \( \frac{p_1 v_1}{m_1 RT_1} \),

\[ \text{i.e. } m_1 = \frac{p_1 v_1}{RT_1} = \frac{(100 \times 10^3) \times 1.45}{(0.287 \times 10^3) \times 298} = 1.695 \text{ kg.} \]

After 21/4 minutes, the mass of the air in the receiver will be,

\[ m_2 = \frac{p_2 v_2}{RT_2} = \frac{(1,500 \times 10^3) \times 1.45}{(0.287 \times 10^3) \times 333} = 22.758 \text{ kg.} \]

\[ \therefore \text{Mass of air compressed per minute, } m = \frac{m_2 - m_1}{2.25} = \frac{22.758 - 1.695}{2.25} = 9.36 \text{ kg} \]

Volume which this air occupies at 100 kPa and 25°C,

\[ v = \frac{mRT}{P} = \frac{9.36 \times (0.287 \times 10^3) \times 298}{(100 \times 10^3)} = 8 \text{ m}^3. \]

Volumetric efficiency of the compressor = \( \frac{v}{9.5} \times 100 = 84.21\% \)

Problem-5 : It is desired to compress 17 m³ of air per minute from 1 bar (100 kN/m²) and 21°C to a delivery pressure of 7 bar (700 kN/m²) in a single-stage, single-acting air compressor. Calculate the power required to drive the compressor and the heat rejected during compression to cooling water if the compression is (a) Isentropic \( (\gamma = 1.4 \text{ for air}) \), and (b) Isothermal.

Neglect clearance effects.
Given: \( p_1 = 1 \text{ bar} = 1 \times 10^5 \text{ Pa}; \quad p_2 = p_2' = 7 \text{ bar} = 7 \times 10^5 \text{ Pa}; \quad \frac{p_2}{p_1} = \frac{7 \times 10^5}{1 \times 10^5} = 7 

(a) Isentropic compression:

Referring to fig. 11-12 and using eqn. (11.6a), isentropic work required per sec.,

\[
W = \frac{\gamma}{\gamma-1} p_1 v_1 \left[ \frac{p_2}{p_1} \right]^{\frac{\gamma-1}{\gamma-1}} - 1 \ 	ext{J/sec.}
\]

\[
= \frac{1.4}{1.4-1} \times (1 \times 10^5)^{1} \times \frac{17}{60} \left[ \frac{7}{1} \right]^{1.4-1} - 1
\]

\[
= \frac{1.4}{0.4} \times (1 \times 10^5) \times \frac{17}{60} \times [1.744 - 1]
\]

\[
= 73,750 \text{ J/sec. or 73,750 W}
\]

\[
\therefore \quad \text{Power required to drive the compressor = 73,750 W i.e. 73.75 kW}
\]

\[
\text{No heat is rejected during isentropic compression.}
\]

(b) Isothermal Compression

Referring to fig. 11-12 and using eqn. (11.7a), isothermal work required/sec.,

\[
W = p_1 v_1 \log_e \left( \frac{p_2}{p_1} \right) = (1 \times 10^5) \times \frac{17}{60} \times \log_e \left( \frac{7}{1} \right)
\]

\[
= (1 \times 10^5) \times \frac{17}{60} \times 1.9459 = 55,080 \text{ J/sec. or 55,080 W}
\]

\[
\therefore \quad \text{Required power input = 55,080 W i.e. 55.08 kW}
\]

In isothermal compression, as the temperature remains constant, there is no change in internal energy and the entire work of compression, i.e. 55.08 kJ/sec. is rejected to jacket cooling water. Heat rejected during isothermal compression = 55.08 kJ/sec.

Problem-6: A two-stage air compressor delivers 145 m³ of free air per hour. The pressure and temperature in the cylinder at the start of compression are 1 bar and 34°C respectively. The diameter of the low-pressure cylinder is twice that of the high-pressure cylinder. The air enters the high-pressure cylinder at a temperature of 40°C and is then compressed to 17.5 bar, the law of compression being \( pv^{1.22} \) = constant for both stages. Neglecting the effects of clearance, estimate: (a) the intercooler pressure, (b) the air power required, and (c) the ratio of cylinder diameters for minimum work making the usual assumptions regarding the intercooler conditions.

The free air conditions are 1.01325 bar and 15°C. Take \( R = 0.287 \text{ kJ/kg K} \) for air.

(a) Since, the diameter of L.P. cylinder is twice that of the H.P. cylinder, the ratio of L.P. to H.P. cylinder volumes will be 4, i.e. \( v_1 = 4v_2 \).

Applying characteristic equations at points of suction in L.P. and H.P. cylinders,
\[
\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{i.e.} \quad \frac{1 \times 4 v_2}{(34 + 273)} = \frac{p_2 \times v_2}{(40 + 273)}
\]

\[\therefore \text{Intercooler pressure, } p_2 = \frac{1 \times 4 \times 313}{307} = 4.078 \text{ bar}\]

(b) Using characteristic equation for free air conditions, \(pv = mRT\)

i.e. \((1.01325 \times 10^5) \times \frac{145}{60} = m \times (0.287 \times 10^5) \times (15 + 273)\)

\[\therefore m = 2.963 \text{ kg of air per minute.}\]

Referring to fig. 11-13, and using eqn. (11.8b) for imperfect intercooling,

Total work required per kg of air, \(W = \begin{cases} \text{Work required in H.P. cylinder} \\ \text{Work required in L.P. cylinder} \end{cases}\)

\[\therefore W = RT_1 \frac{n}{n-1} \left\{ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right\} + RT_2 \frac{n}{n-1} \left\{ \left( \frac{p_3}{p_2} \right)^{n-1} - 1 \right\}
\]

\[= R \frac{n}{n-1} \left[ T_1 \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] + T_2 \left( \frac{p_3}{p_2} \right)^{n-1} - 1 \right\}
\]

\[= 0.287 \times \frac{1.22}{1.22 - 1} \left[ 307 \left( \frac{4.078}{1} \right)^{0.22} - 1 \right] + 313 \left( \frac{17.5}{4.078} \right)^{0.22} - 1 \right\]

\[= 290 \text{ kJ per kg of air}\]

Work done per sec. = \(W \times m = 290 \times \frac{2.963}{60} = 14.32 \text{ kJ/sec.}\)

\[\therefore \text{Air power} = 14.32 \text{ kW}\]

Fig. 11-13. Two stage compression with imperfect intercooling.

Fig. 11-14.
(c) Using eqn. (11.11),

For minimum work, intercooler pressure, \( p_2 = \sqrt{p_1 p_3} = \sqrt{1 \times 17.5} = 4.183 \text{ bar} \)

Since intercooling is perfect, \( T_1 = T_2 \).

\[
\therefore \frac{p_1 v_1}{p_2 v_2} = \frac{V_1}{V_2} = \frac{4.183}{1} = 4.183
\]

\[
\therefore \text{Ratio of cylinder diameters for minimum work,}\n\]

\[
\frac{d_1}{d_2} = \frac{\sqrt{V_1}}{\sqrt{V_2}} = \sqrt[4]{4.183} = 2.045 \quad (\because l_1 = l_2)
\]

Problem-7 : It is desired to compress 16 m\(^3\) of air per minute from 1 bar (100 kPa)
and 294 K to 105 bar (105 MPa). Calculate: (i) the minimum power required to drive
the compressor with two-stage compression and compare it with the power required
for single-stage compression, (ii) the maximum temperature in the two cases. (iii) the heat
to be removed in the intercooler per minute, (iv) the amount of cooling water required
per minute if the inlet and outlet temperatures of cooling water to and from the intercooler
are 15°C and 40°C. Assume the value of index for compression process to be 1.35
for both cases. Also assume proper intercooler pressure for minimum work and perfect
intercooling. Take \( R = 0.287 \text{ kJ/kg K} \) and \( k_p = 1.0035 \text{ kJ/kg K} \) for air.

Given : \( p_1 = 1 \text{ bar} = 1 \times 10^5 \text{ Pa}; \ p_3 = 105 \text{ bar} = 10.5 \times 10^5 \text{ Pa}; \)

\[
\frac{p_3}{p_1} = \frac{10.5 \times 10^5}{1 \times 10^5} = 10.5
\]

Using equation (11.11) for maximum efficiency or minimum work, intercooler pressure,

\[
p_2 = \sqrt{p_1 p_3} = \sqrt{1 \times 10.5} = 3.241 \text{ bar} = 3.241 \times 10^5 \text{ Pa}
\]

\[
\therefore \frac{p_2}{p_1} = \frac{3.241 \times 10^5}{1 \times 10^5} = 3.241
\]

Referring to fig. 11-15 and using eqn. (11.12a),

Minimum work required per sec. for two-stage compression,

![Fig. 11-15. Two-stage compression with perfect intercooling.](image)

![Fig. 11-16. Single-stage compression.](image)
\[ W = 2 \times \frac{n}{n-1} \times p_1v_1 \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] \text{ J/sec.} \]

(where, \( p_1 \) is pressure in pascals (Pa) and \( v_1 \) is volume of air compressed in \( m^3 \) per sec.)

\[ = 2 \times \frac{1.35}{1.35 - 1} \times (1 \times 10^5) \times \frac{16}{60} \left[ \left( \frac{3.241}{1} \right)^{1.35 - 1} - 1 \right] = 73,170 \text{ J/sec. or 73,170 W} \]

Minimum power required for two-stage compression = 73,170 W i.e. 73.17 kW

For single-stage compression (fig. 11-16), the work required/sec. is given by

\[ W = \frac{n}{n-1} p_1v_1 \left[ \left( \frac{p_3}{p_1} \right)^{n-1} - 1 \right] \text{ J/sec.} \]

(where \( v_1 \) is the volume of air taken in per sec. in \( m^3 \) and \( p_1 \) is inlet pressure and \( p_3 \) is final pressure)

\[ = \frac{1.35}{1.35 - 1} \times (1 \times 10^5) \times \frac{16}{60} \left[ \left( \frac{10.5}{1} \right)^{1.35 - 1} - 1 \right] = 86,250 \text{ J/sec. or 86,250 W} \]

Minimum power required to drive the compressor for single-stage compression = 86,250 W i.e. 86.25 kW

(ii) For single-stage compression (fig.11-16), the maximum absolute temperature is \( T_3 \).

\[ \frac{T_3}{T_1} = \left( \frac{p_3}{p_1} \right)^{n-1} \]

\[ \Rightarrow T_3 = T_1 \times \left( \frac{p_3}{p_1} \right)^{n-1} = 294 \left( \frac{10.5}{1} \right)^{1.35 - 1} = 541 \text{ K} \]

\[ \Rightarrow \text{ Maximum temperature with single-stage compression} = 541 - 273 = 268^\circ \text{C.} \]

For two-stage compression (fig. 11-15), the maximum absolute temperature is \( T_2' \).

\[ \frac{T_2'}{T_1} = \left( \frac{p_2}{p_1} \right)^{n-1} \]

\[ \Rightarrow T_2' = T_1 \times \left( \frac{p_2}{p_1} \right)^{n-1} = 294 \times \left( \frac{3.241}{1} \right)^{1.35} = 398.72 \text{ K} \]

\[ \Rightarrow \text{ Maximum temp. with two-stage compression} = 398.72 - 273 = 125.72^\circ \text{C} \]

(iii) Now, mass of air compressed per min.,

\[ m = \frac{p_1v_1}{RT_1} = \frac{(1 \times 10^5) \times 16}{287 \times 294} = 18.96 \text{ kg/min.} \]
Referring to fig. 11-15 and using eqn. (11.10),

Heat rejected by air to the intercooler water per min. with two-stage compression
\[ = mk_p \left( T_2' - T_2 \right) \text{kJ/min.} \]
\[ = 18.96 \times 1.0035 \times (398.72 - 294) = 1,995 \text{kJ/min.} \]

Heat gained by cooling water in the intercooler per min.
\[ = m_f \times 4.187 \times (t_2 - t_1) \text{kJ/min.} \]
\[ = m_f \times 4.187 \times (40 - 15) = 1,995 \text{kJ/min.} \]

Heat rejected by air/min. = Heat gained by cooling water/min.
\[ \text{i.e. } 1,995 = m_f \times 4.187 \times (40 - 15) \]
\[ \therefore m_f = 19.06 \text{kg/min. (mass of cooling water per min.)} \]

Problem-8: A two-stage, single-acting air compressor for a Diesel engine runs at 250 r.p.m. and takes in 6 m³ of air per minute at a pressure of 1 bar and temperature of 15°C. It delivers the air at 70 bar and compression is carried out in each cylinder according to the law \[ pV^{1.3} = \text{constant} \]. Assuming complete intercooling and mechanical efficiency of 80 per cent, determine the minimum power required to drive the compressor. Calculate also the cylinder diameters and common stroke, if the average piston speed is 170 metres per minute. Neglect clearance effects and wire-drawing losses.

Referring to fig. 11-17, and using eqn. (11.12b),

Minimum indicated power
\[ = \frac{2n}{n-1} p_1 v_1 \left[ \left( \frac{p_3}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{watts} \]
\[ = \frac{2 \times 1.3}{1.3 - 1} \times (1 \times 10^5) \times \frac{6}{60} \left[ \left( \frac{70}{1} \right)^{\frac{1.3 - 1}{1.3}} - 1 \right] \]
\[ = 54,773 \text{ W} = 54.773 \text{ kW} \]
\[ \therefore \text{Power required to drive the compressor (i.e. brake power) taking into consideration the given mechanical efficiency.} \]
\[ = \frac{54,773}{0.8} = 68,466 \text{ kW} \]

i.e. Minimum brake power required to drive the compressor = 68,466 kW

Stroke, \( I = \frac{\text{mean piston speed per min.}}{\text{no. of strokes per min.}} \]
\[ = \frac{170}{2 \times 250} = 0.34 \text{ m or 34 cm.} \]

L.P. Cylinder stroke volume in m³, \( v_1 = \frac{6}{250} = \frac{\pi}{4} (d_1)^2 \times I \)
\[ \text{i.e. } \frac{6}{250} = \frac{\pi}{4} (d_1)^2 \times 0.34 \]
\[ \therefore (d_1)^2 = 0.0899 \]
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... L.P. cylinder dia, \( d_1 = \sqrt{0.0899} = 0.3 \) m, or 30 cm.

Now, for perfect-intercooling, \( p_1 v_1 = p_2 v_2 \),
i.e. \( p_1 \times d_1^2 = p_2 \times d_2^2 \) \((l_1 = l_2)\)
\[
\begin{align*}
\therefore \left( \frac{d_2}{d_1} \right)^2 &= \frac{p_1}{p_2} = \frac{p_1}{\sqrt{p_1 p_3}} = \frac{\sqrt{\frac{1}{70}}}{1} = 0.1196 \\
\therefore \left( \frac{d_2}{d_1} \right) &= \sqrt{0.1196} = 0.346 \\
\therefore \text{H.P. cylinder dia.}, \ d_2 &= d_1 \times 0.346 = 0.3 \times 0.346 = 0.1038 \text{ m, or 10.38 cm.}
\end{align*}
\]

Problem-9: A four-stage air compressor works between the pressures of 1 bar and 140 bar and the index of compression in each stage is 1.23. The temperature at the start of compression in each cylinder is 48°C and the intercooler pressure are so chosen that the work is divided equally between the stages. If the clearance effect be neglected, estimate:

(i) the volume of free air at a pressure of 1.01325 bar and temperature of 15°C which would be dealt with per kW-hour, and (ii) the isothermal efficiency referred to 15°C,

Take \( R = 0.287 \) kJ/kg K for air.

Now, \( \frac{p_5}{p_1} = \frac{140 \times 10^5}{1 \times 10^5} = \frac{140}{1} \)

(i) Using eqn. (11.18) for minimum work, four-stage work required per kg of air,
\[
W = \frac{4n}{n-1} R T_1 \left[ \left( \frac{p_5}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]
\]
\[
= \frac{4 \times 1.23}{0.23} \times 0.287 \times 321 \left[ \left( \frac{140}{1} \right)^{0.23} - 1 \right] = 528.145 \text{ kJ/kg of air.}
\]
But, one kW-hour = 1 \times 3,600 = 3,600 kJ

\therefore \text{Mass of air dealt with, } m \text{ per kW-hour} = \frac{3,600}{528.145} = 6.816 \text{ kg}

Corresponding volume of this (6.816 kg) air at 1.01325 bar and 15°C,
\[
V = \frac{m R T}{p} = \frac{6.816 \times 0.287 \times 10^3 \times 288}{1.01325 \times 10^5}
\]
\[
= 5.56 \text{ m}^3 \text{ of free air/kW-hour}
\]
(ii) Isothermal work required referred to 15°C per kg of air,
\[
W = R T_1 \log_e \left( \frac{p_5}{p_1} \right)
\]
\[
= 0.287 \times 288 \times \log_e \left( \frac{140}{1} \right) = 408.454 \text{ kJ per kg of air}
\]
Isothermal efficiency = Work required when compression is isothermal
Actual work required to be done on the air

\[ \frac{408.454}{528.145} = 0.7734, \text{ or } 77.34\% \]

Problem-10: The following particulars apply to a two-stage, single-acting air compressor:
- Stroke = 28.5 cm; Low pressure cylinder = 23 cm;
- Final pressure = 25 bar; Intermediate pressure = 5 bar;
- Temperature of air leaving the intercooler = 35°C

If the air drawn in the compressor is at 1 bar and 15°C, find the power expended (used) in compressing air when running at 350 r.p.m. Assume law of compression as \( pv^{1.3} \) = constant for both stages.

Swept volume of the L.P. cylinder, \( v_1 = \frac{\pi}{4} d^2 \times l = \frac{\pi}{4} \left( \frac{23}{100} \right)^2 \times \frac{28.5}{100} = 0.01189 \text{ m}^3 \).

Referring to fig. 11-18, and considering polytropic compression 1-2',
\[ p_1 v_1^n = p_2 v_2'^n \]
\[ \therefore v_2' = \left( \frac{p_1}{p_2} \right) \times v_1 = \left( \frac{1}{5} \right) \times 0.01189 = 0.00344 \text{ m}^3. \]

Again, \( \frac{T_2'}{T_1} = \left( \frac{p_2}{p_1} \right)^{n-1} \)
\[ \therefore T_2' = T_1 \times \left( \frac{p_2}{p_1} \right)^{n-1} = (15 + 273) \times \left( \frac{5}{1} \right)^{0.3} = 417.3 \text{ K} \]

Now, \( \frac{p_2 v_2}{T_2} = \frac{p_2 v_2'}{T_2'} \)
\[ \therefore v_2 = T_2 \times \frac{v_2'}{T_2'} = (35 + 273) \times \frac{0.00344}{417.3} = 0.00254 \text{ m}^3. \]

Referring to fig. 11-18 and using eqn. (11.8a) for two-stage compression with imperfect intercooling,
Total work required per cycle, \( W = \text{Work done in L.P. cylinder} + \text{Work done in H.P. cylinder} \)
\[ = p_1 v_1 \frac{n}{n-1} \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] + p_2 v_2 \frac{n}{n-1} \left[ \left( \frac{p_3}{p_2} \right)^{n-1} - 1 \right] \]
Work required per second = \( W \times \frac{N}{60} \) = 4,800 \times \frac{350}{60} = 28,000 J/s or 28,000 W

\[ \therefore \text{Power required} = 28 \text{ kW} \]

**Problem-11:** A three-stage air compressor works between pressures of 100 kPa (100 kN/m\(^2\)) and 5 MPa (5 MN/m\(^2\)). For one \( m^3 \) of air taken in, calculate:

(a) the work required assuming conditions to be for maximum efficiency,

(b) the isothermal work required between the same pressure limits,

(c) the work required if the compressor was one-stage only,

(d) the percentage saving in work input to the compressor by using three-stages instead of single-stage, and

(e) the isothermal efficiency.

In parts (a) and (c), assume that the index of compression in each stage, \( n = 1.3 \).

Referring to fig. 11-19 and using eqn. (11.16b) for maximum efficiency or minimum work,

\[
W = \frac{3n}{n-1} \times p_1 v_1 \left[ \left( \frac{p_4}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ Joule.}
\]

\[
= \frac{3 \times 1.3}{1.3 - 1} \times (100 \times 10^3) \times 1 \left[ \left( \frac{5,000}{100} \right)^{1.3 - 1} - 1 \right]
\]

\[ = 4,56,300 \text{ J} \]

Alternative solution for work required in terms of \( p_1 \) and \( p_2 \).

Referring to fig. 11-19, pressures \( p_1, p_2, p_3 \) and \( p_4 \) are in geometric progression for maximum efficiency or minimum work.

Then, \( p_4 = p_1 \times (R)^m \) where, \( R = \) pressure ratio of each stage and \( m = \) number of stages

i.e. \( 5,000 = 100 \times (R)^3 \) \[ R = \frac{50}{69} = 3.69 \]

Using eqn. (11.16a), minimum work required for one \( m^3 \) of air taken in,

\[
W = \frac{3n}{n-1} p_1 v_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]
\]
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\[
\frac{3 \times 1.3}{1.3 - 1} \times (100 \times 10^3) \times 1 \left[ (3.69)^{1.3 - 1} - 1 \right] = 4,56,300 \text{ J (same as before)}
\]

(b) Referring to fig. 11-20 and using eqn. (11-23), isothermal work required for one \( m^3 \) of air,

\[ W = p_1 V_1 \log_3 \left( \frac{p_4}{p_1} \right) \]

\[ = (100 \times 10^3) \times 1 \times \log_3 \left( \frac{5,000}{100} \right) = 3,91,210 \text{ J} \]

(c) Referring to fig. 11-20, single-stage work required for one \( m^3 \) of air,

\[ W = \frac{n}{n - 1} p_1 V_1 \left[ \left( \frac{p_4}{p_1} \right)^{n-1} - 1 \right] \]

\[ = \frac{1.3}{1.3 - 1} \times (100 \times 10^3) \times 1 \times \left[ \left( \frac{5,000}{100} \right)^{1.3 - 1} - 1 \right] = 6,37,000 \text{ J} \]

(d) Saving in work input to the compressor by adopting three-stages instead of single-stage compression = 6,37,000 - 4,56,300 = 1,80,700 J.

\[ \therefore \text{Percentage saving in work input} = \frac{1,80,700}{6,37,000} = 0.2835 \text{ or 28.35%} \]

(e) Isothermal efficiency = \frac{\text{the isothermal work input}}{\text{the actual work input}} = \frac{3,91,210}{4,56,300} = 0.856 \text{ or 85.6%} \]

11.7 Effect of Clearance on Volumetric Efficiency

The clearance space is provided in an actual compressor to safeguard the piston from striking the cylinder head or cylinder cover. The events taking place in a reciprocating compressor with clearance are the same as those taking place in a compressor without clearance. All the air compressed in the cylinder at the end of the compression stroke will not be discharged from it but some air will be left in the clearance space at the end of the delivery stroke 2-3 (fig. 11-21). The high pressure air, thus, left in the clearance space will re-expand along the curve 3-4 to the suction pressure \( (p_1) \) before the suction valve can open and the suction starts again. The volume of air drawn into the cylinder without clearance is equal to the dis-
placement volume corresponding to full stroke $v_a$. However with clearance, the volume of fresh air drawn into the cylinder is only $v_a$ corresponding to stroke 4-1.

Thus, the effect of clearance in a compressor is to decrease the amount of fresh air that can be drawn into the cylinder during the suction stroke. Therefore, there is a considerable reduction in the volumetric efficiency of the compressor due to clearance. In practice the clearance volume is limited to, two or three per cent of the displacement or swept volume ($v_a$) of the piston.

11.7.1 Expression for work done: Assuming that the value of the index $n$ for expansion curve 3-4 (fig. 11-21) is the same as that for the compression curve 1-2, and making use of eqn. (11.2),

Net work required per cycle, $W = \text{area } 1-2-3-4 = \text{area } 1-2-5-6 \text{ minus area } 5-3-4-6$

$$W = \frac{n}{n-1} p_1 v_a \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] - \frac{n}{n-1} p_1 v_a \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right]$$

$$= \frac{n}{n-1} p_1 (v_1 - v_4) \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right]$$

$$= \frac{n}{n-1} p_1 v_a \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] \text{ Joule per cycle.} \quad \text{(11.28)}$$

where pressures are measured in $N/m^2$ or $p_a$ and volumes in $m^3$.

It may be noted that eqn. (11.28) is same as eqn. (11.2).

This shows that the work required to compress and deliver the same volume of air, $v_a$ (volume of fresh air drawn in the cylinder) in a compressor with clearance, is same as that required in a compressor without clearance.

In other words, the introduction of clearance does not theoretically increase work of compression as the work done in compressing the clearance space air will be regained during the expansion of the clearance air from $v_3$ to $v_4$ at the beginning of the suction stroke.

Net work required in $= W \times \frac{N}{60}$ J/sec. or $W$ where $N$ = no. of cycles per min.

Indicated power of the compressor $= \frac{\text{watts}}{1,000} \text{ kW} \quad \text{(11.29)}$

If $p_1 v_a$ in eqn. (11.28) is substituted by $mRT_1$, then net work required per kg of air may be written as

$$W = \frac{n}{n-1} R T_1 \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] \text{ Joule.}$$

which is same as eqn. (11.4).

Net work required in J/sec.

$$= W \times \text{mass of air delivered per second J/sec. or } W \quad \text{(11.30)}$$
11.7.2 Expression for Volumetric Efficiency: Let $v_c$ and $v_s$ be the clearance volume and swept volume respectively of the compressor, $p_2$ = pressure in N/m$^2$ = $p_a$ of air in the clearance space, $p_1$ = pressure in N/m$^2$ = $p_a$ of clearance air at the end of expansion, and $n$ is the index of expansion.

Referring to fig. 11-21, Volume of clearance air at the end of re-expansion,

$$v_4 = v_3 \left( \frac{p_2}{p_1} \right)^\frac{1}{n} = v_c \left( \frac{p_2}{p_1} \right)^\frac{1}{n}$$

The volume of fresh charge of air, $v_a = v_1 - v_4$

$$v_a = v_1 - v_c \left( \frac{p_2}{p_1} \right)^\frac{1}{n} = v_s + v_c - v_c \left( \frac{p_2}{p_1} \right)^\frac{1}{n} \quad [ \therefore v_1 = v_s + v_c ]$$

$$= v_s - v_c \left[ \left( \frac{p_2}{p_1} \right)^\frac{1}{n} - 1 \right]$$

Volumetric efficiency, $\eta_v = \frac{v_1 - v_4}{v_s} = \frac{v_a}{v_s}$

$$\frac{v_s - v_c \left( \frac{p_2}{p_1} \right)^\frac{1}{n} - 1}{v_s} = 1 - \frac{v_c \left( \frac{p_2}{p_1} \right)^\frac{1}{n} - 1}{v_s} \quad \ldots (11.31)$$

Thus, the volumetric efficiency depends upon the pressure ratio, $p_2 / p_1$ and the clearance ratio, $v_c / v_s$. Volumetric efficiency decreases as the pressure ratio increases. It also decreases as clearance volume (as a percentage of swept volume) increases or volumetric efficiency decreases as the clearance ratio increases.

Problem-12: A compressor has 20 cm bore and 30 cm stroke. It has a linear clearance of 15 cm. Calculate the theoretical volume of air taken in per stroke when working between pressures of 1 bar and 7 bar. The index of compression and expansion is same and its value is 1.25.

Referring to fig. 11-22,

Clearance volume $v_c = v_3$

$$v_3 = \text{Area of cylinder} \times \text{linear clearance}.$$  

$$\frac{\pi}{4} d^2 \times \text{linear clearance}$$

$$\frac{\pi}{4} (0.2)^2 \times \frac{1.5}{100} = 0.000471 \text{ m}^3.$$  

Considering polytropic expansion (3-4),

$$p_3 v_3^n = p_4 v_4^n$$

$$\therefore v_4 = v_3 \left( \frac{p_3}{p_4} \right)^\frac{1}{n}$$

Fig. 11-22. Single-stage air compressor with clearance.
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\[ v_4 = 0.000471 \times \left( \frac{7}{1} \right)^{1.25} = 0.002233 \text{ m}^3. \]

Now, \( v_1 = \) stroke volume, \( v_s + \) clearance volume = \( \frac{\pi}{4} d^2 \times l + v_3 \)

\[ = \frac{\pi}{4} (0.2)^2 \times 0.3 + 0.000471 = 0.009891 \text{ m}^3. \]

Theoretical volume of air taken in per stroke,

\[ v_a = v_1 - v_4 = 0.009891 - 0.002233 = 0.007658 \text{ m}^3. \]

Problem-13 : The clearance volume in a single cylinder single-acting reciprocating compressor is 5% of the swept volume. Air is drawn in at 100 kPa (100 kN/m\(^2\)) and 311 K. Compression and expansion curves follow the law \( pv^{1.2} = C. \) The delivery pressure is 700 kPa (700 kN/m\(^2\)) and the atmospheric pressure and temperature are 101.325 kPa and 15°C respectively. Estimate :

(a) the volumetric efficiency,
(b) the volumetric efficiency referred to atmospheric conditions, and
(c) the required work input per kg of air.

Take \( R = 0.287 \text{ kJ/kg K} \) for air.

(a) Referring to fig. 11-23,

Since, the index \( n \) is the same for both compression and expansion,

\[ \frac{T_2}{T_1} = \frac{T_3}{T_4} \]

But, \( T_3 = T_2 \)

\[ \therefore T_4 = T_1 = 311 \text{ K} \]

Now, \( p_3(v_3)^n = p_4(v_4)^n \)

\[ \therefore v_4 = v_3 \left( \frac{p_3}{p_4} \right)^{\frac{1}{n}} \]

\[ = 0.05v_s \left( \frac{700}{100} \right)^{\frac{1}{1.2}} \]

\[ = 0.05v_s \times 5.06 = 0.253v_s \]

Fig. 11-23. Single-stage air compressor with clearance.

\[ v_a = v_1 - v_4 = (1.05 - 0.253)v_s = 0.797v_s \]

Volumetric efficiency, \( \eta_v = \frac{v_a}{v_s} = \frac{0.797v_s}{v_s} \)

\[ = 0.797 \text{ or } 79.7\% \]

Alternatively, using eqn. (11-31),

Volumetric efficiency, \( \eta_v = 1 - \frac{v_c}{v_s} \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \)
where \( p_2/p_1 \) is pressure ratio, \( v_c/v_s \) is the clearance ratio and \( n \) is the index of expansion.

\[
\eta_v = 1 - \frac{0.05v_s}{v_s} \left[ \frac{\left( \frac{700}{100} \right)^{1.2}}{1.2 - 1} \right] = 0.797 \text{ or } 79.7\% \text{ (same as before)}
\]

(b) Corresponding volume, \( v_s \) at 101.325 kPa and 15°C (atmospheric conditions)

\[
= 0.797v_s \times \frac{100}{101.325} \times \frac{288}{311} = 0.7283v_s
\]

\[
\text{[Volumetric efficiency referred to atmospheric conditions]} = \frac{\text{volume of free air drawn in per stroke}}{\text{swept volume per stroke}} = \frac{0.7283v_s}{v_s} = 0.7283 \text{ or } 72.83\%
\]

(c) Using eqn. (11.30) or eqn. (11.4), required work input per kg of air,

\[
W = RT_1 \frac{n}{n-1} \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ Joule}
\]

\[
= 287 \times 311 \times \frac{1.2}{1.2 - 1} \left[ \left( \frac{700}{100} \right)^{1.2 - 1} - 1 \right] = 2,05,110 \text{ J/kg of air}
\]

Note: Work input/kg of air is not affected by clearance.

Problem-14: A double-acting, single cylinder air compressor runs at 100 r.p.m. The air is compressed from an initial pressure of 100 kPa (1 bar) to a delivery pressure of 750 kPa (7.5 bar). The stroke volume is 0.15 m³ and law of compression and expansion is \( pv^{1.25} = C \). If the clearance volume is 1/18th of the stroke volume, calculate:

(a) the volume of air taken in per stroke, and

(b) the indicated power of the compressor.

(a) Referring to fig. 11-24 and using eqn. (11.31), Volumetric efficiency,

\[
\eta_v = 1 - \frac{v_c}{v_s} \left( \frac{p_2}{p_1} \right)^n - 1
\]

\[
= 1 - \frac{18v_s}{v_s} \left[ \left( \frac{750}{100} \right)^{1.25} - 1 \right]
\]

\[
= 0.7773 \text{ or } 77.73\%
\]

Volume of air taken in per stroke (or cycle), \( v_s \)

\[
= \text{stroke volume} \times \text{volumetric efficiency}, \eta_v
\]

\[
= 0.15 \times 0.7773 = 0.1166 \text{ m}^3/\text{cycle or stroke}
\]
(b) Using eqn. (11.28), work required per cycle,

\[
W = \frac{n}{n-1} p_a \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{Joule}
\]

\[
= \frac{1.25}{1.25 - 1} \times (100 \times 10^3) \times 0.1166 \left[ \left( \frac{750}{100} \right)^{\frac{1.25 - 1}{1.25}} - 1 \right]
\]

= 28,920 Joule per cycle.

\[\therefore \text{Work required in Joules per second}\]

\[= W \times \frac{N}{60} \text{ (where, } N = \text{no. of cycles per min.} = 2 \times 100 = 200)\]

\[= 28,920 \times \frac{200}{60} = 96,400 \text{ J/sec. or } 96,400 \text{ W}\]

\[\therefore \text{Indicated power of the compressor } = 96,400 \text{ W or } 96.4 \text{ kW.}\]

**Problem-15:** A single-cylinder air compressor compresses air from 1 bar to 7 bar. The clearance volume is 2 litres and law of compression and expansion is \(pv^{1.2} = \text{constant}\). If the volumetric efficiency of the compressor is 80 per cent, determine: (i) the stroke volume and (ii) the cylinder dimensions. Assume stroke of the piston equal to the diameter of the cylinder.

Here \(p_1 = 1\) bar, \(p_2 = 7\) bar, \(v_c = 2\) litres, \(n = 1.2\) and \(\eta_v = 80\) per cent.

Using eqn. (11.31), Volumetric efficiency, \(\eta_v = 1 - \frac{v_c}{v_s} \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right]\)

Substituting the values, we get, \(0.8 = 1 - \frac{2}{v_s} \left[ \left( \frac{7}{1} \right)^{\frac{1}{1.2}} - 1 \right]\)

i.e. \(0.8 = 1 - \frac{2}{v_s} [4.06]\) or \(\frac{2}{v_s} = \frac{1 - 0.8}{4.06} = \frac{0.2}{4.06}\)

\[\therefore v_s = \frac{2 \times 4.06}{0.2} = 40.6 \text{ litres} = 40.6 \times 10^3 \text{ cm.}\]

Now, \(v_s = \frac{\pi}{4} \times d^2 \times d\) \((\because l = d)\)

i.e. \(40.6 \times 10^3 = \frac{\pi}{4} d^3\)

\[\therefore d = \sqrt[3]{\frac{40.6 \times 10^3 \times 4}{\pi}} = \sqrt[3]{51,693.5} = 37.25 \text{ cm.}\]

\[\therefore \text{Cylinder diameter, } d = 37.25 \text{ cm, and piston stroke, } l = d = 37.25 \text{ cm.}\]

**Problem-16:** A single-acting, single-cylinder air compressor is to deliver 15 kg of air per minute at 7 bar from suction conditions of 1 bar and 30°C. The clearance volume is 5% of the stroke volume and the law of compression and expansion is \(pv^{1.2} = \text{constant}\).
The compressor is direct coupled to a four-cylinder, four-stroke petrol engine which runs at 1,400 r.p.m. with a brake mean effective pressure (b.m.e.p.) of 7 bar. If the stroke and bore are equal for both engine and compressor and mechanical efficiency of the compressor is 85 per cent, calculate the required cylinder dimensions of the compressor and engine.

Take \( R = 0.287 \) kJ/kg K for air.

Referring to fig. 11-25, and considering polytropic expansion 3-4,

\[
p_3 (v_3)^n = p_4 (v_4)^n
\]

\[
\therefore v_4 = v_3 \left( \frac{p_3}{p_4} \right)^\frac{1}{n}
\]

\[
= 0.05 v_a \times \left( \frac{7}{1} \right)^\frac{1}{1.2}
\]

\[
= 0.253 v_s
\]

\[
v_a = v_1 - v_4
\]

\[
= 1.05 v_s - 0.253 v_s
\]

\[
= 0.797 v_s
\]

Now, \( p v_a = mRT \)

(where \( v_a \) is the volume/min.)

\[
\therefore v_a = \frac{mRT}{p} = \frac{15 \times 0.287 \times (30 + 273)}{1 \times 10^2} = 13.044 \text{ m}^3/\text{min.} = \frac{13.044}{1,400} \text{ m}^3/\text{stroke.}
\]

\[
\therefore 0.797 v_s = \frac{13.044}{1,400} \quad \therefore v_s = 0.0117 \text{ m}^3 \text{ or } 11,700 \text{ cm}^3
\]

Now, \( v_s = \frac{\pi}{4} d^2 \times d = 11,700 \text{ cm}^3 \) \( (\therefore l = d) \)

\[
\therefore d^3 = 14,897 \quad \therefore d = \sqrt[3]{14,897} = 24.61 \text{ cm}
\]

\[
\therefore \text{Piston stroke, } l = d = 24.61 \text{ cm (compressor)}
\]

Using eqn. (11.30), W.D. per sec. = \( n \frac{mRT}{n-1} \sqrt{\left( \frac{p_2}{p_1} \right)^{n-1} - 1} \) J

(where \( m \) is the mass of air delivered/sec.)

\[
= \frac{1.2}{0.2} \times \frac{15}{60} \times (0.287 \times 10^3) \times 303 \left[ \left( \frac{7}{1} \right)^{\frac{1}{1.2}} \right] - 1
\]

\[
= 49,959 \text{ J/sec. or } 49,959 \text{ W}
\]

Indicated power of the compressor = 49,959 kW
Brake power of the compressor

\[
\text{Indicated power} \quad \frac{49.959}{0.85} = 58.775 \text{ kW (input power to compressor)}
\]

But, brake power of engine,

\[
= p_m \times l \times a \times \text{no. of working cycles of the engine per sec.} \times \text{no. of cylinders}
\]

where \( p_m \) = brake mean effective pressure in \( p_a \),

\( l \) = length of engine piston stroke in metre, and

\( a \) = area of piston in \( m^2 \).

i.e. \((58.775 \times 10^3) = (7 \times 10^5) \times d \times 0.7854 \times 0.1318 \times 4\)

\[
\therefore d^3 = 0.002291 \quad \therefore d = 0.1318 \text{ m or } 13.18 \text{ cm, and } l = 13.18 \text{ cm.}
\]

Problem-17: A single-cylinder, single-acting air compressor is required to deliver 7 m³ of free air per minute at a mean piston speed of 200 m per minute. The air is to be compressed from an initial pressure of 1 bar to a delivery pressure of 7.5 bar and the index of compression and expansion may be assumed to be 1.25. Assuming the stroke of piston to be 1.5 times the bore of the cylinder, clearance volume to be 1/18th of the swept volume per stroke, and suction pressure and temperature to be equal to the atmospheric air pressure and temperature, find the volumetric efficiency, bore, stroke and speed of the compressor.

Referring to fig. 11-26.

Since the index \( n \) is the same for both compression and expansion,

\[
\frac{T_2}{T_1} = \frac{T_3}{T_4}
\]

But, \( T_3 = T_2 \quad \therefore T_4 = T_1 \)

Volume of free air drawn in per stroke \( V_a \)

\[
V_a = V_1 - V_4
\]

Now, \( p_3 \times V_3^n = p_4 \times V_4^n \)

\[
\therefore V_4 = V_3 \left( \frac{p_3}{p_4} \right)^n = \frac{V_3 \left( 7.5 \right)^{\frac{1}{1.25}}}{18} = 0.2782 \text{ } V_3
\]

(Where, free air pressure or atmospheric pressure is taken as 1 bar)

Now, \( V_1 = V_3 + \frac{1}{18} \times V_3 = 1.0555 \times V_3 \)

\[
V_a = V_1 - V_4 = 1.0555 \times V_3 - 0.2782 \times V_3 = 0.7773 \times V_3
\]

Volumetric eff., \( \eta_v = \frac{\text{Volume of free air drawn in per stroke, } V_a}{\text{Stoke volume, } V_s} \)

\[
= \frac{0.7773 \times V_3}{V_3} = 0.7773 \text{ or } 77.73\%
\]
Now, \( v_s = \frac{\pi}{4} d^2 \times l = \frac{\pi}{4} d^2 \times 1.5d = 1.18d^3 \)

Also mean piston speed per min. = 200 m/min. = \( 2l \times \text{r.p.m.} \)

\[ \therefore \text{Speed of compressor, r.p.m.} = \frac{200}{2l} = \frac{200}{2 \times 1.5d} = \frac{200}{3d} \]

Volume of air delivered in m\(^3\) per minute, \( v_a = (v_f - v_4) \times \text{r.p.m.} = 7 \)

i.e. \( 0.7773 v_s \times \text{r.p.m.} = 7 \)

i.e. \( 0.7773 (1.18d^3) \times \frac{200}{3d} = 7 \)

\[ \therefore d = 0.3382 \text{ m or 33.82 cm.} \text{ Thus, stroke, } l = 1.5 \times 0.3382 = 0.5073 \text{ m or 50.73 cm.} \]

Speed of the compressor, r.p.m. = \( \frac{200}{3d} = \frac{200}{3 \times 0.3382} = 197 \text{ r.p.m.} \)

Problem-18 : The following particulars refer to a two-stage, single-acting air compressor:

Capacity, 5 m\(^3\) per minute measured under free air conditions of 15°C and 1.01325 bar; Delivery pressure, 17 bar; Pressure during suction stroke, 0.98 bar; Temperature of air at the start of compression in each stage, 30°C; Clearance volume of low-pressure cylinder is 6% of the swept volume; Index of compression and expansion, 1.25 throughout; Speed, 125 r.p.m.

Assuming that the intercooler pressure is so chosen that theoretically the work is shared equally between the two cylinders, find : (a) the indicated power, and (b) the dimensions of the low-pressure cylinder if the bore is equal to the stroke. Take \( R = 287 \text{ J/kg K} \) for air.

(a) Referring to fig.11-27,

![Fig. 11-27. Two-stage air compressor with clearance.](image)

Mass of air dealt with,

\[ m = \frac{pv}{RT} = \frac{(1.01325 \times 10^5) \times 5}{287 \times 288} = 6.129 \text{ kg per minute.} \]

From eqn. (11.11) for work to be shared equally between the two cylinders, intercooler pressure is given by

\[ p_2 = \sqrt{p_1 p_3} = \sqrt{0.98 \times 17} = 4.08 \text{ bar.} \]

Now, \( \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \)

\[ \therefore T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = 303 \left( \frac{4.08}{0.98} \right)^{\frac{0.25}{1.25}} = 303 \times 1.33 = 403 \text{ K.} \]

Work done in L.P. cylinder per kg of air is given by

\[ W = \frac{n}{n-1} R (T_2 - T_1) = \frac{1.25}{0.25} \times 287 (403 - 303) = 1,435,000 \text{ J.} \]
Since, the work is equally shared between the two cylinders, 
The total work done per kg of air = 1,43,500 × 2 = 2,87,000 J 
Work required in Joule per sec. = work done per kg of air × mass of air per sec. 
= 2,87,000 × \frac{6.129}{60} = 29,320 J/sec. or 29,320 W 
∴ Indicated power = 29.32 kW.

(b) Let stroke volume of L.P. cylinder, \( v_1 - v_3 = 100 \text{ m}^3 \), then \( v_3 = 6 \text{ m}^3 \) and \( v_1 = 106 \text{ m}^3 \) (fig. 11-27).

Mass of air dealt with per stroke, for a single-acting compressor with 100 \text{ m}^3 stroke volume, \( m = m_1 - m_3 \). 

\[
m_1 = \frac{p_1 v_1}{RT_1} = \frac{0.98 \times 10^5 \times 106}{287 \times 303} = 119.46 \text{ kg} \text{ (mass of air compressed)}
\]

\[
m_3 = \frac{p_3 v_3}{RT_3} = \frac{4.08 \times 10^5 \times 6}{287 \times 403} = 21.17 \text{ kg} \text{ (mass of air left in the cylinder)}
\]

∴ \( m = m_1 - m_3 = 119.46 - 21.17 = 98.29 \text{ kg per 100 m}^3 \text{ stroke volume and assuming that the temperature remains constant during delivery, i.e. from point 2 to 3,} \)

Actual stroke volume per minute = \( \frac{100}{98.29} \times 6.129 = 6.236 \text{ m}^3/\text{min.} \)

Now, stroke volume per min. = \( \frac{\pi}{4} d^2 \times d \times \text{r.p.m.} \)

i.e. 6.236 = \( \frac{\pi}{4} d^2 \times d \times 125 \) (the bore being equal to the stroke)

∴ \( d^3 = 0.0636 \) ∴ \( d = 0.3991 \text{ m or 39.91 cm}, \text{ and } l = d = 39.91 \text{ cm}. \)

Problem-19 : A two-stage air compressor delivering air at 17.5 bar has a clearance volume of 4% of the swept volume. The atmospheric conditions are 1.01 bar and 18°C, and at the start of compression the pressure in the cylinder is 1 bar. The temperature at the start of compression in each stage is 30°C, and the intercooler pressure is 4.04 bar. The law of compression and expansion for both stages is \( pv^{1.25} = \text{constant} \).

Find : (a) the volumetric efficiency referred to atmospheric conditions,
(b) the work required per kg of air delivered by the compressor, and
(c) the isothermal efficiency referred to isothermal compression from atmospheric temperature and pressure. \( (R = 0.287 \text{ kJ/kg K for air}) \).

Referring to fig. 11-28,

Let the stroke volume, \( v_1 - v_3 = 100 \text{ units}, \)
then \( v_3 = 4 \text{ units} \) and \( v_1 = 104 \text{ units}. \)

\[
T_1 = T_5 = (30 + 273) = 303 \text{ K}
\]

(∵ cooling is perfect)
(a) Considering polytropic compression 1-2 in L.P. cylinder.

\[ \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \]

\[ \therefore T_2 = T_1 \times \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = 303 \times \left( \frac{4.04}{1} \right)^{\frac{1.25-1}{1.25}} = 400.5 \text{ K} \]

Now, \( p_1 v_1^n = p_2 v_2^n \) or \( \frac{v_1}{v_2} = \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} \)

\[ \therefore v_2 = \frac{v_1 \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}}}{\left( \frac{p_2}{p_1} \right)^{\frac{1}{n}}} = \frac{104}{4.04 \frac{1}{1.25}} = 30.03 \text{ units} \]

\[ \therefore \text{Volume of air delivered} = v_2 - v_3 = 34.03 - 4 = 30.03 \text{ units measured at } 4.04 \text{ bar and } 400.5 \text{ K}, \]

also \( \frac{p v}{T} \) (measured conditions) = \( \frac{p_2 v_2}{T_2} \) (free air conditions)

\[ \therefore \text{Volume of free air (at atmospheric conditions i.e. at } 1.01 \text{ bar and } 291 \text{ K}) \text{ delivered} \]

\[ = 30.03 \times 4.04 \times \frac{291}{1.01 \times 400.5} = 87.28 \text{ units}. \]

\[ \text{Volumetric efficiency referred} \]
\[ \frac{\text{to atmospheric conditions}}{\text{Swept volume of L.P. cylinder per stroke}} \]

\[ = \frac{87.28}{100} = 0.8728 \text{ or } 87.28\% \]

(b) Work done per kg of air in L.P. cylinder

\[ = \frac{n}{n-1} R (T_2 - T_1) = \frac{1.25}{1.25 - 1} \times 0.287 (400.5 - 303) = 139.91 \text{ kJ} \]

Considering polytropic compression 5-6 in H.P. cylinder, \( \frac{T_6}{T_5} = \left( \frac{p_6}{p_5} \right)^{\frac{n-1}{n}} \)

\[ \therefore T_6 = T_5 \left( \frac{p_6}{p_5} \right)^{\frac{n-1}{n}} = 303 \left( \frac{17.5}{4.04} \right)^{\frac{0.25}{1.25}} = 303 \times 1.341 = 406.3 \text{ K} \]

Work done per kg of air in H.P. cylinder

\[ = \frac{n}{n-1} R (T_6 - T_5) = \frac{1.25}{1.25 - 1} \times 0.287 (406.3 - 303) = 148.24 \text{ kJ} \]

Therefore, total work required per kg of air

\[ = \text{work required in L.P. cylinder} + \text{work required in H.P. cylinder} \]

\[ = 139.91 + 148.24 = 288.15 \text{ kJ per kg of air.} \]

(c) Isothermal work done per kg of air for the same range of pressure and compression from atmospheric pressure and temperature,
\[ W = R \times T_{atm} \times \log_e \left( \frac{p_2}{p_{atm}} \right) \]

\[ = 0.287 \times 291 \times \log_e \left( \frac{17.5}{1.01} \right) = 0.287 \times 291 \times 2.852 = 238.19 \text{ kJ per kg of air} \]

\[ \text{Isothermal efficiency} = \frac{\text{Work done when compression is isothermal per kg of air}}{\text{Actual work done on the air per kg of air}} \]

\[ = \frac{238.19}{288.15} = 0.8266 \text{ or 82.66%} \]

11.8 Actual Indicator Diagram of a Single-Stage Air Compressor

Actual air compressor differs from the ideal compressor in many respects, because the cylinder, the valves and the piston, store and release energy in the form of heat and further because air is throttled through the valves. An indicator diagram of an actual air compressor is shown in fig. 11-29 superimposed upon an ideal one (shown dotted) for the same compressor. The air pressure is to be raised from atmospheric pressure \( p_1 \) to a receiver pressure \( p_2 \).

The ideal indicator diagram is drawn for isothermal compression of air in the cylinder and for isothermal expansion of the clearance air. In actual compressor, isothermal compression is not obtained, instead polytropic compression is obtained, i.e. the compression falls between the adiabatic (isentropic) and the isothermal, but it is closer to adiabatic as indicated by compression line \( 1' - 2' \) in fig. 11-29. The value of index \( n \) varies from 1.25 to 1.35. In actual compressor, the clearance air will become cooler than the cylinder walls with which it is in contact hence it will receive heat during the latter part of the expansion process. The expansion line in actual compressor lies between an adiabatic and an isothermal, but generally it is closer to isothermal. The actual expansion line obtained will have shape and position similar to line \( 3' - 4' \) instead of line \( 3 - 4 \).

In ideal air compressor, the intake valve will open as soon as the clearance air pressure has decreased to atmospheric but actually the intake valve does not open until the pressure drops a little below atmospheric. After the valve is open and air is in motion, there are generally several oscillations of the valve and the air column as indicated by the wavy portion on the suction line \( 4' - 1' \). The discharge valve does not open till the pressure attained is slightly above that in the receiver; and after opening, it behaves much like the intake valve for similar reason. The discharge line actually obtained will have a shape and position similar to line \( 2' - 3' \) instead of line \( 2 - 3 \).

There is always some leakage past the valves, piston rings and piston rod packings in the actual compressor; and greater the leakage the larger will be the work required for any given mass of air delivered.

The net result of the actual compressor has been to deliver a smaller mass of
Air than that handled by the ideal compressor and to require an expenditure of work in excess of that of the ideal one as shown by greater area.

11.9 Reciprocating Compressed Air Motor

Air motor is in effect a reversed air compressor. The compressed air to be used in an air motor is taken from the compressor reservoir (receiver). The most common form of compressed air motor is the cylinder and double-acting piston type. The air is admitted into the motor cylinder through a mechanically operated inlet valve and drives the piston forward but after a portion of the stroke of the piston has been performed, the air supply is cut-off and the stroke is completed under decreasing pressure as the air expands in the cylinder. After the expansion stroke is completed, the air which has done the work is allowed to escape into atmospheric through a mechanically operated discharge valve. The return stroke is performed by compressed air acting on the other side of the piston. A motor of this type works like a reciprocating double-acting steam engine.

The important application of air motor is the use in mines where use of electric motor is dangerous. There will be a fall in the pressure of air due to friction in the pipe, the fall being greater, the greater the distance of the air motor from the compressor reservoir (receiver).

Figure 11-30 shows the pressure-volume diagram for a compressed air motor (air engine) in which clearance is neglected. Air is admitted at high pressure $p_1$ from 4-1, cut off takes place at point 1. From 1 to 2 air expands from pressure $p_1$ to atmospheric pressure $p_2$ and expansive work is done. The law of expansion is polytropic, i.e. $pv^n = C$. Exhaust takes place from 2 to 3.

The work done per cycle by the air motor with no clearance is given by area 4-1-2-3.

\[
\text{Work done per cycle} = p_1v_1 + \frac{p_1v_1 - p_2v_2}{n-1} - p_2v_2
\]

\[
= \frac{n p_1v_1 - p_1v_1 + p_1v_1 - p_2v_2 - np_2v_2 + p_2v_2}{n-1}
\]

\[
= \frac{n}{n-1} \left[ p_1v_1 - p_2v_2 \right]
\]

\[
= \frac{n}{n-1} p_2v_2 \left[ \frac{p_1v_1}{p_2v_2} - 1 \right] \text{Joule}
\]

But for polytropic expansion 1-2,

\[
p_1v_1^n = p_2v_2^n \quad \therefore \quad \frac{v_1}{v_2} = \left( \frac{p_1}{p_2} \right)^\frac{1}{n}\]
Substituting the value of \( \frac{v_1}{v_2} \) in the above equation, we get, work done by air,

\[
W = \frac{n}{n-1} p_2 v_2 \left[ \left( \frac{p_1}{p_2} \right)^{\frac{n-1}{n}} - 1 \right] \text{ Joule per cycle} \quad \text{.. (11.32)}
\]

The fig. 11-31 shows the pressure-volume diagram in which air is admitted at high pressure \( p_1 \) from 5 to 1, cut-off takes place at point 1, expansive work is done from 1 to 2 and release takes place (before the atmospheric pressure reached) from 2 to 3 and then exhaust takes place from 3 to 4.

The work done per cycle by the air motor, with no clearance, is given by area 5-1-2-3-4.

Work done by air motor per cycle

\[
= \text{area } 5-1-2-3-4
\]

\[
= p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n-1} - p_3 v_3 \quad \text{.. (11.33)}
\]

where, pressures are measured in Pa and volumes in m³.

**Problem-20:** Air at a pressure of 9 bar enters the cylinder of a compressed air motor; the supply is cut-off at \( 1/4 \)th stroke and the air expands according to the law \( p v^{13} = \) constant. Neglecting the effect of clearance, find the temperature of air at the end of expansion when the temperature of the compressed air is 25°C, and find the volume for a single-acting air motor to develop indicated power of 5 kW at a speed of 260 r.p.m. when the exhaust is at 1.05 bar. Also find the air motor cylinder dimensions assuming bore = stroke.

Assume \( R \) for air = 0.287 kJ/kg K.

Referring to fig. 11-32, and considering polytropic expansion 1-2,

\[
\frac{T_1}{T_2} = \left( \frac{v_2}{v_1} \right)^{n-1} = (\eta)^{n-1}
\]

\[
\therefore \quad T_2 = \frac{T_1}{(\eta)^{n-1}} = \frac{25 + 273}{(4)^{0.3}} = 197 \text{ K}
\]

i.e. \( t_2 = 197 - 273 = -76^\circ \text{C} \), temperature of air at the end of expansion.

It may be noted that the temperature of air at the end of expansion is -76°C which is much below the freezing point. When the temperature falls below the freezing point, the moisture in the air is frozen and, the snow formed may seriously
interfere with the working of the discharge (exhaust) valve of the air motor.

The most satisfactory method of preventing too low a temperature at the end of expansion is to heat the air (before the air enters the motor cylinder) at constant pressure by passing it through a suitable pre-heater. By pre-heating the air, not only the freezing effect is prevented but the volume of air is increased, the volume being proportional to the absolute temperature and consequently a large proportion of the heat expended (used up) in the heater is converted into work in the motor cylinder, i.e. the work done in the motor cylinder, for the consumption of same mass of air increases, when the air is pre-heated.

Now, \[ p_1 v_1^n = p_2 v_2^n \] \[ p_2 = \frac{p_1}{(v_2/v_1)^n} = \frac{p_1}{(r)^n} = \frac{9}{(4)^{1/3}} = 1.485 \text{ bar} \]

Using eqn. (11.33), Work done/stroke = \[ p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n-1} - p_3 v_3 \]

\[ = 10^5 v_2 \left[ \frac{p_1 v_1}{v_2} + \frac{9}{4} - 1.485 \right] = 3,75,000 v_2 \text{ J per stroke} \]

Indicated power (work done in J/sec or W) = Work done per stroke \( \times \frac{N}{60} \)

i.e. \[ 5 \times 10^3 = 3,75,000 \times v_2 \times \frac{260}{60} \] (for a single-acting motor)

\[ v_2 = 0.003077 \text{ m}^3. \]

But, \[ v_2 = \frac{\pi}{4} d^2 \times l \] \[ \therefore 0.003077 = \frac{\pi}{4} \times d^3 \] \[ \therefore d^3 = \frac{0.003077}{0.7854} = 0.0039177 \]

\[ \therefore \text{Dia. of cylinder, } d = 0.0039177^{1/3} = 0.1576 \text{ m or 15.76 cm and stroke, } l = d = 15.76 \text{ cm.} \]

11.10 Classification of Air Compressors

The compressors can be mainly classified as, depending on the type of mechanism, the nature of their motion, the flow of fluid, etc. Fig. 11-33 shows the classification of compressors.

In a positive displacement compressor, displacement of a fixed amount of fluid (positively contained during its passage through the machine), takes place. In that type, work changes are effected by the displacement of a boundary, i.e., by pressure and volume changes. All reciprocating compressors are positive displacement type, while rotary compressors may be positive displacement or non-positive displacement type.

In reciprocating positive displacement compressors, the displacement of a boundary reduces the volume and increases the pressure. In rotary positive displacement compressors the increase in pressure is by backflow of fluid as in Root's blower, or both by variation in volume and by backflow as in vane type blower.
In non-positive displacement or steady flow compressors, the fluid is continuously in a steady-flow through the machine, undergoing changes in pressure primarily by means of dynamic effects. In this type of compressor, work changes are effected by means of velocity changes. Therefore, operating speed affects directly the operating efficiency of a non-positive displacement compressor; while operating speed, within reasonable limits, has little effect on the efficiency of a positive displacement compressor.

Reciprocating compressors have the following advantages over non-positive displacement rotary compressors:

- Capacity of delivering higher pressures,
- Greater flexibility in capacity and pressure range,
- Somewhat higher efficiency for pressure ratios > 2, and
- Less susceptibility to change in gas composition and density.

Reciprocating compressors are suitable for delivering small quantities of air at high pressure. The maximum free air delivery is limited to 300 m³/min but the pressure may be as high as 1,000 bar. It is a low speed machine due to the inertia of the reciprocating parts.

Because of large quantities of air at relative low pressure (3 to 10 bar) required in gas turbine plants, reciprocating compressors, though they have above mentioned advantages are not employed.

Positive displacement rotary or displacement blowers are also not suitable for gas turbine plants as they, in general, are noisy, subject to wear, and also of limited capacity compared with the non-positive displacement or steady flow rotary compressors (centrifugal and axial flow compressors).

Non-positive displacement rotary or steady flow compressors, Centrifugal and axial flow compressors, offer the following advantages:

- Handle large quantities of air or gas, at relatively lower pressure,
- Being high speed machines, they are small in size,
- Provide uniform delivery without requiring a large receiver,
- Deliver clean air as there are fewer sliding parts requiring lubrication,
- Present no balancing problems,
- Lower maintenance expenses, and
- Less operating attention.
11.11 Rotary Air Compressors

Depending on their pressure ratio, \( r_p = \frac{P_2}{P_1} \), rotary air compressors may be classified as follows:

*Fans* (\( r_p = 1 \) to 1.1) – The main purpose of a fan is to move air or to circulate air through the ducts of an air conditioning system and to supply low-pressure air blast. As the fluid passing through the unit does not suffer appreciable change in density, it can be treated as incompressible like water. Principally, fans are axial-flow compressors without casing around the impeller periphery.

*Blowers* (\( r_p = 1.1 \) to 4) and compressors (\( r_p > 3 \)) – If compressibility effect is considerable, it is to be taken into account while designing the unit and the unit is known as *blower* or a *compressor* proper depending on the pressure ratio. Blower may be positive displacement rotary compressor (Root's blower, lysholm, vane type, screw type, etc.) or non-positive displacement rotary compressor (centrifugal compressor).

The purposes of a blower is to furnish forced draught to furnaces, besemer convertors, cupolas, etc.

The purpose of compressor (centrifugal or axial flow) is to supply compressed air in large quantity, such as in gas turbine plants.

11.12 Non-positive Displacement Rotary or Steady Flow Compressors

*Centrifugal* and *axial flow* compressors come under this heading. In all forms of turbo-machinery, whether expanders (called turbines or centrifugal and axial flow compressors), work changes are effected by means of velocity changes instead of by displacement changes of a boundary, such as with a piston. If the air enters at the centre axially and flows radially outward towards the circumference, the compressor is known as *centrifugal compressor*. On the other hand if the air flows parallel to the shaft, it is called *axial flow compressor*.

11.12.1 Centrifugal Air Compressors: A centrifugal compressor, mainly, consists of impeller and diffuser. The impeller consists of an impeller disc and impeller vanes, attached on the impeller disc radially, forming radial diverging passages as shown in fig. 11-34(a). The impeller rotates with the shaft at high speed and air is drawn into the impeller eye in an axial direction. The air then flows radially outwards through the impeller passages due to centrifugal force, and kinetic energy in imparted to the air with some static pressure rise as shown in fig. 11-34(b). The remainder of the pressure rise is obtained in the diffuser. The diffuser which is stationary, consists of a number of fixed diverging passages. The air leaves the impeller tip with high velocity and enters the diffuser. The diffuser, in its fixed diverging passages, reduces the high velocity. Thus, by the diffusion process or deceleration of air in the diffuser, kinetic energy is converted into pressure energy.

The flow from the diffuser is collected in a spiral passage from which it is discharged from the compressor.

Single-stage centrifugal compressor can develop a pressure ratio as high as 4 : 1. Pressure ratio as high as 10 : 1 can be developed with the help of multi-stage centrifugal compressors.

The impeller may be single or double sided. The impeller illustrated in fig. 11-34(a) is single sided one. The single sided impeller sucks in air from one side only. A double
sided impeller consists of vanes on both sides and air is drawn in from both sides. A double sided impeller is used often when larger flows are to be handled.

The centrifugal compressor has the following advantages as compared with axial flow compressor:

(i) It is more rugged.
(ii) It is simpler.
(iii) It is less expensive.
(iv) It is smaller in length.
(v) It attains higher pressure ratio per stage.
(vi) It is not liable to loss of performance by the effects of deposits left on the surfaces of flow passages of air when working in a contaminated atmosphere.
(vii) It is able to operate efficiently over a wider range of mass flow at any particular speed.

However, it suffers from disadvantages of large frontal area, lower maximum efficiency, and less adaptability to multi-staging.

Centrifugal compressor is mainly used in, (i) Oil refinery, (ii) Petrochemical plants, (iii) Natural gas transmission system, (iv) Refrigeration plants, (v) Supercharging of Petrol and Diesel engines, etc.

11.12.2 Axial Flow Air Compressors: Axial flow compressor consists of a rotor and a casing or stator. Blades mounted on the rotor
are moving blades (rotor blades) and blades fixed on the inner face of the casing are known as stationary blades (stator blades). Air enters the blades axially and also leaves them in the axial direction as shown in fig. 11-35(a).

An axial flow compressor is designed according to the reaction principle (diffusion in both rotor and stator). Thus, the rise in pressure through the stage (rotor and stator) is in general, attributable to both blade rows, moving as well as fixed. As a measure of the extent to which the rotor itself contributes to this pressure rise, the term "degree of reaction" is used. Thus, degree of reaction is defined as the ratio of pressure rise in rotor to the total pressure rise.

A single-stage compressor (one row of moving blades and one row of fixed blades) does not give appreciable pressure ratio. Axial flow compressors are mostly multi-stage compressors. Pressure ratio which can be produced per stage of an axial flow compressor is $1 : 2$.

Figure 11-35(a) represents a three-stage, axial flow compressor. The moving blades, receive the air and increase its velocity, and also act as a diffuser to increase the pressure, while the fixed blades continue the diffuser action. Figure 11-35(b) shows the increase in pressure of air during its passage through rotor blades (R) and stator blades (S).

Axial flow type compressors are given preference over the centrifugal type in the application of aircraft and industrial gas turbine power plants. This is because axial flow compressor has a higher efficiency, less frontal area, and is capable of producing higher pressure ratio on a single shaft by increasing the number of stages. However, the axial flow compressors run at lower speeds, their weight is greater, have higher starting torque, are sensitive to any deposit formation on blades, and are complicated as compared with centrifugal compressors.

**Tutorial - 11**

1. (a) Estimate the amount of work required for compression of one kg of air for single-stage compression.
   (b) What are the uses of high pressure air?
   (c) A single-stage, single-acting air compressor deals with 85 m$^3$ of free air per hour at 1 01325 bar and 15°C. The pressure and temperature in the cylinder during the suction stroke remains constant at 1 bar and 40°C respectively. The index of compression is 1.22 and the delivery pressure is 5.5 bar. If the mechanical efficiency is 85%, find, neglecting clearance volume, the power required to drive the compressor.

   $[6.107 \text{ kW}]$

2. A single-acting, single-cylinder, air compressor has a cylinder diameter 18 cm and a stroke of 25 cm. Air is drawn into the cylinder at 102 kPa (102 kN/m$^2$), 288 K. It is then compressed isentropically to 632 kPa (632 kN/m$^2$). Find: (a) the power required to drive the compressor if its speed is 120 r.p.m; and (b) the mass of air compressed per hour. Take $\gamma = 1.4$ and $R = 0.287 \text{ kJ/kg K}$ for air. Neglect clearance effects.

   $[3.112 \text{ kW}; (b) 56.54 \text{ kg/hr.}]$

3. It is desired to compress 15 m$^3$ of air per minute at 100 kPa (100 kN/m$^2$) and 297 K to 700 kPa (700 kN/m$^2$) in a single-stage, single-acting air compressor. Calculate the power required to drive the compressor if the compression is:
   (a) Isothermal,
   (b) Polytropic with index $n = 1.3$,
   (c) Isentropic.

   Take $\gamma = 1.4$ for air. Neglect clearance effects.

   $[(a) 48.648 \text{ kW}; (b) 61.533 \text{ kW}; (c) 65.1 \text{ kW}]$
4. (a) A single-stage, single-acting air compressor is belt driven from an electric motor at 400 r.p.m. The cylinder diameter is 15 cm and stroke is 18 cm. The air is compressed from 1.05 bar to 7.5 bar and the law of compression is \( pv^{1/3} = \text{constant} \). Find the power of the motor, if the transmission efficiency is 97% and the mechanical efficiency of the compressor is 90%. Neglect the clearance effects.

\[6.354 \text{ kW}\]

(b) A single-cylinder, double-acting air compressor is of 24 kW. The law of compression is \( pv^{1/2} = \text{constant} \). The air is compressed from 1 bar to 8 bar. The compressor runs at 200 r.p.m. and average piston speed may be taken as 160 m/min. Find the dimensions of the cylinder required. Neglect the effect of clearance.

\[d = 21.47 \text{ cm}; \quad l = 0.4 \text{ m}\]

5. (a) Define volumetric efficiency of a compressor.

(b) Find the diameter and stroke of a single-stage, double-acting air compressor from the following data:
- Capacity: 20 m³ of free air per minute at atmospheric pressure and temperature.
- Delivery pressure: 7.5 bar.
- Inlet pressure: 1 bar.
- Inlet temperature: 32°C.
- Atmospheric pressure: 1.01325 bar.
- Atmospheric temperature: 15°C.
- Index of compression: 1.3.
- Stroke: bore = 0.9 : 1.

Also find the indicated power of the compressor. Neglect clearance effects.

\[d = 36.99 \text{ cm}; \quad l = 33.291 \text{ cm}; \quad 91.753 \text{ kW}\]

6. A two-stage, single-acting air compressor draws air at 1.05 bar and temperature of 16°C. The air is compressed in the L.P. cylinder to a pressure of 5.25 bar. After compression, the air is cooled in the intercooler at constant pressure to a temperature of 30°C, before being taken to H.P. cylinder from which it is delivered at 20 bar. The compression is carried out in each stage according to the law \( pv^{1.35} = \text{C} \). Find the indicated work required in kJ to compress one kg of air. Take \( R = 0.287 \text{ kJ/kg K} \) for air. Neglect clearance.

\[304.92 \text{ kJ}\]

7. What are the conditions for obtaining maximum efficiency in the case of a two-stage reciprocating air compressor?

A two-stage, single-acting air compressor deals with 5.75 m³ of air per minute under atmospheric conditions. It delivers air at 40 bar. The compressor runs at 300 r.p.m. The stroke is equal to the L.P. cylinder diameter. Assuming complete intercooling and compression in each stage according to the law \( pv^{1.35} = \text{C} \) and mechanical efficiency of 85 per cent, calculate the cylinder diameters and minimum power required to drive the compressor. Assume atmospheric conditions as 1.01325 bar and 15°C. Neglect clearance.

\[d_{L.P.} = 29 \text{ cm}; \quad d_{H.P.} = 11.56 \text{ cm}; \quad 53.758 \text{ kW}\]

8. A two-stage, single-acting air compressor takes in 3 m³ of air per minute at atmospheric pressure of 1 bar and temperature of 288 K. It delivers air at 35 bar (3.5 MPa). The compression is carried out in each cylinder according to the law \( pv^{1.35} = \text{C} \). Assuming complete intercooling and mechanical efficiency of 80 per cent, calculate the minimum power required to drive the compressor and the heat carried away from the intercooler per minute by cooling water. If the inlet and outlet temperatures of the cooling water are 15°C and 40°C, calculate the amount of cooling water required per minute. Also compare the volumes of the two cylinders. Neglect clearance effects. Take \( R = 0.287 \text{ kJ/kg K} \) and \( K_p = 1.0035 \text{ kJ/kg K} \) for air.

\[26.688 \text{ kW}; \quad 447.93 \text{ kJ/min.;} \quad 4.28 \text{ kg/min.;} \quad 5.916\]

9. What is the object of compressing air in stages?

Derive an expression for the minimum work required to compress and deliver 1 kg of air in a two-stage compressor. State carefully the assumptions made.

10. What are the advantages of multi-stage compression?

Determine the minimum driving power required for three-stage compression from 1.05 bar to 42 bar if the delivery is 4.5 m³ of free air per min. Compression in each stage is carried out according to law \( pv^{1.3} = \text{constant} \). Suction temperature is 21°C and mechanical efficiency is 90%. Take free air conditions as 1.01325 bar and 15°C. The air compressor is single-acting. Neglect clearance.

\[41346 \text{ kW}\]

11. A three-stage, single-acting air compressor with perfect intercooling deals with 23 m³ of air per minute at 1.01325 bar and 288 K. The L.P. cylinder suction pressure and temperature are 1 bar and 305 K and the final delivery pressure is 25 bar. If the stage pressures are in geometric progression and index of compression in each stage is 1.35, find the power required to drive compressor. Also find the heat rejected to the intercoolers per minute and isothermal efficiency of the compressor. Neglect clearance. Take \( R = 0.287 \text{ kJ/kg K} \) and \( K_p = 1.0035 \text{ kJ/kg K} \) for air.

\[152.3 \text{ kW}; \quad 2760 \text{ kJ/min.;} \quad 86.86%\]
12. What is meant by the term "perfect cooling"?

A three-stage, single-acting compressor has perfect intercooling. The pressure and temperature at the end of suction stroke in the L.P. cylinder is 1.01325 bar and 15°C respectively. The delivery pressure is 70 bar. Compression in each stage is carried out according to \( pv^{1.25} = C \), and 6.5 m\(^3\) of free air (1.01325 bar and 15°C) are delivered per min. If the work done is minimum, calculate:

(i) the L.P. and I.P. delivery pressures,
(ii) the ratio of cylinder volume, and
(iii) the indicated power of the compressor.

Neglect clearance

\[(i) 14.158 \text{ bar}, 17.059 \text{ bar}; (ii) 16.835 : 4.103 : 1; (iii) 68.73 \text{ kW}\]

13. A three-stage, single-acting air compressor works between 1.05 bar and 42 bar. For one cubic metre of air taken in, calculate:

(i) the indicated work required in kJ for maximum efficiency,
(ii) the isothermal work required in kJ between the same pressure limits,
(iii) the indicated work required if the compressor were of one-stage only,
(iv) the isothermal efficiency of the compressor, and
(v) the percentage saving in work required due to using three-stages instead of one.

In parts (i) and (iii), assume the index of compression, \( n = 1.25 \).

Neglect clearance.

\[(i) 438.64 \text{ kJ}; (ii) 387.34 \text{ kJ}; (iii) 572.78 \text{ kJ}; (iv) 88.3\%; (v) 23.42\%\]

14. (a) Explain the effect of clearance on the performance of air compressor.

(b) Explain that volumetric efficiency depends on clearance volume and pressure ratio for a single-stage compressor.

The clearance volume in a single-stage, single-acting air compressor is 5% of the swept volume. Air is drawn in at constant pressure of 1 bar and temperature of 43°C. Compression and expansion follow the law \( pv^{1.25} = \text{constant} \). The delivery pressure is 6.5 bar and atmospheric pressure and temperature are 1.01325 bar and 15°C respectively. Estimate:

(a) the volumetric efficiency,
(b) the volumetric efficiency referred to atmospheric conditions, and
(c) the work required per kg of air.

Take \( R = 0.287 \text{ kJ/kg K} \) for air.

\[(a) 82.65\%; (b) 74.34\%; (c) 205.87 \text{ kJ/kg}\]

15. A single-acting, single-cylinder air compressor compresses air from 1 bar to 6.5 bar. Compression and expansion follow the law \( pv^{1.25} = \text{constant} \). The clearance volume is 1 litre. If the volumetric efficiency of the compressor is 80%, calculate the stroke volume. If the ratio of diameter of cylinder to stroke of piston is 1.5, calculate diameter of the cylinder and stroke of the piston.

\[0.0173 \text{ m}^3; 24.52 \text{ cm}; d = 36.78 \text{ cm}\]

16. A single-acting, single-cylinder air compressor runs at 100 r.p.m. The air is compressed from 1 to 8 bar. The stroke volume is 0.125 m\(^3\) and the law of compression and expansion is \( pv^{1.3} = \text{constant} \). If the clearance volume is 5% of the stroke volume, calculate:

(i) the volumetric efficiency,
(ii) the volume of air taken in per minute, and
(iii) the indicated power of the compressor.

\[(i) 80.245\%; (ii) 10.031 \text{ m}^3/\text{min}.; (iii) 44.55 \text{ kW}\]

17. In an air compressor, show that cylinder clearance does not affect the theoretical work required to compress and deliver one kg of air, provided that delivery and suction pressures remain constant, and that the indices of compression and expansion have the same value.

A single-stage, single-acting air compressor is required to deliver 6 m\(^3\) of free air per minute at a mean piston speed of 165 m per min. The air is to be compressed from an initial pressure of 1.05 bar to delivery pressure of 7 bar and index of compression and expansion is assumed to be 1.3. Assuming stroke of piston to be 1.25 times the bore of the cylinder, clearance volume to be 1/15th of the swept volume per stroke and suction pressure and temperature to be equal to atmospheric pressure and temperature, find:

(i) the volumetric efficiency, (ii) the speed, and (iii) the bore and stroke.

\[(i) 78\%; (ii) 191.64 \text{ r.p.m.}; (iii) d = 34.47 \text{ cm}, l = 43.08 \text{ cm}\]
18. A single-stage, double-acting air compressor has a stroke volume of 0.06 m³ and a clearance volume of 0.003 m³. Find its volumetric efficiency referred to atmospheric conditions (1.01325 bar and 15°C) and the mass of air delivered per hour when the speed is 200 r.p.m. The suction pressure is 0.92 bar and suction temperature is 50°C and the delivery pressure is 6 bar. Also determine the indicated power of the compressor. Assume compression and expansion law as $pv^{1.35} = \text{constant}$. Take $R = 0.287$ kJ/kg K for air.

\[ 68.77\%; 1,213.9 \text{ kg/hr}; 75.476 \text{ kW} \]

19. A single-acting, single-stage air compressor is required to compress 5 kg of air per minute from 0.95 bar and 30°C to a pressure of 7.6 bar. The clearance volume is 5% of the stroke volume and the index of both the expansion and compression curves is 1.25. If the stroke and bore are equal and compressor runs at 120 r.p.m., find the size of the cylinder.

Take $R = 0.287$ kJ/kg K for air.

\[ d = l = 39.53 \text{ cm} \]

20. A two-stage, single-acting air compressor has to deal with 3 m³ of air per minute under atmospheric conditions (1.01325 bar and 15°C) at 220 r.p.m. and delivers at 85 bar. Assuming complete intercooling between the stages, find the minimum power required to drive the compressor, the cylinder diameters and the common stroke. Assume a piston speed of 165 m per min, mechanical efficiency of compressor as 80% and volumetric efficiency of 85% for each stage. Compression in each cylinder is carried out according to the law $pv^{1.3} = \text{constant}$. Neglect clearance.

\[ 36.553 \text{ kW}; d_{\text{LP}} = 23.34 \text{ cm}; d_{\text{HP}} = 7.712 \text{ cm}; l = 37.5 \text{ cm} \]

21. In a two-stage, single-acting air compressor, the delivery pressure is 17.5 bar and the suction pressure is 1 bar. The temperature at the start of compression in each stage is 30°C and the index of compression in each stage is 1.25. The clearance volume of the low-pressure cylinder is 5 per cent of the swept volume and the diameter of low-pressure cylinder is 0.8 of the stroke. The mass of air delivered by the compressor is 5 kg/min. and the intercooler pressure has the ideal value. Find:

(a) the bore of the L.P. cylinder if the speed of the compressor is 110 r.p.m., and
(b) the indicated power of the compressor.

The expansion of the clearance air may be assumed to follow the same law as that for compression.

\[ (a) 35.59 \text{ cm}; (b) 23.988 \text{ kW} \]

22. The following particulars apply to a two-stage, single-acting air compressor: Stroke = 25.4 cm; Low-pressure cylinder diameter = 29.2 cm; Final pressure = 25 bar; Intermediate pressure = 5 bar; Temperature of air leaving the intercooler = 35°C.

If the volume of air drawn in the compressor and measured at 1 bar and 15°C, is 80% of the low-pressure cylinders swept volume, find the power expended (used) in compressing the air when running at 250 r.p.m. Assume law of compression as $pv^{1.3} = C$ in each stage.

\[ 22.88 \text{ kW} \]

23. Plot actual and theoretical indicator diagrams of a single-stage reciprocating air compressor. Discuss why they differ.

24. Describe briefly the working of an air motor (compressed air engine), Where it is used?

Air at pressure of 8.5 bar enters the cylinder of an air motor; the supply is cut-off at quarter stroke and expands according to the law $pv^{1.35} = \text{constant}$. Neglecting the effect of clearance, find the temperature at the end of expansion when the initial temperature of air is 30°C. Calculate also the cylinder volume for a single-acting air motor to develop 3.7 kW power at a speed of 250 r.p.m., when exhaust is at 1.05 bar.

Take $R = 0.287$ kJ/kg K for air.

\[ -86.5^\circ \text{C}; 2,604.9 \text{ cm}^3 \]

25. What are rotary compressors? Differentiate between blowers and compressors. What are axial flow compressors?

26. (a) How are compressors classified?
(b) Distinguish between:
   (i) Reciprocating and rotary compressors, and
   (ii) Positive displacement and non-positive displacement compressors.

27. (a) Differentiate between a fan, a blower, and a compressor.
(b) What is the difference between centrifugal and axial flow compressors?

28. Sketch and describe the operation of a single-stage centrifugal compressor.

29. (a) What are the advantages and disadvantages of centrifugal compressors?
(b) Sketch and describe the operation of an axial flow air compressor.
(c) What are the advantages and disadvantages of axial flow compressors as compared with centrifugal compressors?