9.1 Heat Engines

Heat engines may be divided into two main classes, according to where the combustion of fuel takes place. In one class, the combustion of fuel takes place outside the cylinder, and such an engine is called external combustion engine. The most common examples of this class are steam engines and steam turbines, where the working medium is steam. In an external combustion engine the power is produced in two stages. The energy released from the fuel in the furnace of the boiler is first utilized to evaporate water in a boiler and then the steam so produced is made to act on the piston of the steam engine or on the blades of the steam turbine producing power. When the combustion of fuel takes place inside the engine cylinder so that the products of combustion directly act on the piston, the engine is known as internal combustion engine. Diesel engine, gas engine and petrol engine are the common examples of this class where the working medium is the products of combustion.

Steam engines were manufactured upto the year 1930 for use as stationary prime-movers, particularly in the textile industry. They are still used for locomotives for railways and now slowly they are being replaced by Diesel locomotives. In addition, they are used on ships where they are slowly being replaced by steam turbines and Diesel engines.

9.2 Steam Engine Plant

As steam engine plant consists essentially of three main units: Boiler, Engine and Condenser. In many cases, particularly in locomotive steam engines, a separate condenser is not provided and the engine exhausts into the atmosphere.

The steam from the boiler is admitted into a steam chest from where it enters the engine cylinder through a valve driven by an eccentric on the engine crankshaft. After expansion in the engine cylinder and doing work on the piston, the steam is exhausted into a condenser where it is condensed and returned as feed water to the boiler, thus, completing the cycle. Nearly all reciprocating steam engines are

Fig. 9.1. Side view of a horizontal simple steam engine.
double-acting, i.e. steam is admitted in turn to each side of the piston and two working strokes are produced during each revolution of the crankshaft.

Figure 9-1 illustrates a simple form of a single-cylinder, horizontal, reciprocating steam engine. The figure shows major principal parts of the engine.

9.3 Classification (Types)

Steam engines may be classified in the following ways:

- Position of the axis of the cylinder: Vertical, Inclined or Horizontal engine.
- According to the action of steam upon the piston: Single-acting or Double-acting engine.
- Number of cylinders used in which steam expands: Single-expansion or Simple engine (total expansion of steam in one cylinder), and Multiple-expansion or compound engine (total expansion of steam in more than one cylinder).
- Method of removal of exhaust steam: Condensing or Non-condensing engine.
- Magnitude of rotative speed: Low, Medium or High speed engine.
- Type of valve used: Slide valve, Corliss valve or Drop valve engine.
- Use or field of application: Stationary, Portable (movable), Locomotive, Marine engine.

9.4 Parts of Steam Engine

The parts of steam engine may be broadly divided into two groups, namely, stationary parts and moving parts.

- Stationary parts: Engine frame, Cylinder, Steam chest, Stuffing box, Crosshead guides and Main bearings.
- Moving parts: Piston and piston rod, Crosshead, Connecting rod, Crankshaft, Flywheel, Slide valve and valve rod, Eccentric and eccentric rod and Governor.

The function of the steam engine parts are as follows:

The engine frame is a heavy casting which supports all the stationary as well as moving parts of the engine and holds in proper alignment. It may rest directly on the engine foundation or upon the engine bed plate fixed on the engine foundation.

The cylinder shown in fig. 9-2 is a cast iron cylindrical hollow vessel in which the piston moves to and fro under the pressure of the steam. Both the ends of the cylinder are closed by covers and made steam-tight.

The steam chest is a closed chamber integral with the cylinder. It supplies steam to the cylinder with the movement of the slide valve.

The stuffing box and gland are fitted on the crank end cover of the cylinder as shown in fig. 9-2 and
their function is to prevent the leakage of steam past the piston rod which moves to and fro.

The piston is a cast iron cylindrical disc moving to and fro in the cylinder under the action of the steam pressure. Its function is to convert the heat energy of the steam into mechanical work. Cast iron piston rings make the piston steam tight in the cylinder and thereby prevent the leakage of steam past the piston.

The crosshead is a link between piston rod and the connecting rod. It guides the motion of the piston rod and prevents it from bending.

The connecting rod helps in converting the reciprocating motion of the piston into rotary motion of the crank. Its one end is connected to the crosshead by means of gudgeon pin or crosshead pin and other end is connected to the crank.

The FPS (fig. 9-3) is the main shaft of the engine and carries on it the flywheel and the eccentric. It is supported on the main bearings of the engine and is free to rotate in them. It is made of mild steel. Crank formed on the crankshaft works on the lever principle and produces rotary motion of the crankshaft.

The slide valve (fig. 9-2) is situated in the steam chest and its function is to admit the steam from steam chest to the cylinder, and exhaust the steam from the cylinder at the proper moment. The valve gets to and fro motion from the eccentric fitted on the crankshaft.

The eccentric is fitted on the crankshaft. The function of eccentric is to convert the rotary motion of the crankshaft into reciprocating motion of the slide valve.

The main bearings support the engine crankshaft and are fitted on the engine frame. The part of the crankshaft which turns in the bearing is called a main bearing journal as shown in fig. 9-3.

The flywheel is a heavy cast iron or cast steel wheel mounted on the crankshaft to prevent the fluctuation of engine speed throughout the stroke and to carry the crank smoothly over the dead centres.

The steam engine governor is a device for keeping the speed of the engine more or less constant at all loads. For this it controls either the quantity or pressure of the steam supplied to the engine according to the load on the engine.

9.5 Working of a Simple, Double-acting, Condensing Steam Engine

The function of a steam engine is to convert the heat energy of steam into mechanical work. The pressure of the steam acts on the piston and moves it to and fro in the cylinder. It is necessary to have some method of converting this to and fro motion of the piston into a rotary motion, since the rotary motion can be conveniently transmitted from the engine to any other driven machine. This to and fro motion of the
The piston is converted into rotary motion with the help of connecting rod and crank of the steam engine.

The steam (fig. 9-4) is first admitted to the cover end (left hand side) of the cylinder from the steam chest when steam admission port is uncovered (opened) by the D-slide valve, while the exhaust steam (which has done work on the piston) on the crank end (right hand side) of the cylinder passes at the same time into a vessel, called condenser (not shown in the figure) through the steam port and the exhaust port as shown in fig. 9-4. This steam admitted to the cover end exerts pressure on surface of the piston and pushes it to crank end (right hand side) of the cylinder.

At the end of this stroke, fresh steam from the steam chest is again admitted by the D-slide valve to the crank end of the cylinder (when admission steam port is opened), while the exhaust steam on the cover end of the cylinder passes at the same time into the condenser through steam port and exhaust port. Thus, the steam at the cover end exhausts while that at the crank end pushes the piston back to its original position.

The D-slide valve gets to and fro motion from the eccentric fitted on the crankshaft.

Thus, two working strokes are completed and the crankshaft turns by one revolution, i.e., the engine is double-acting. These operations are repeated.

When the exhaust steam is exhausted to atmosphere, the engine is known as non-condensing engine.

The motion of the piston and piston rod moves the crosshead, connecting rod, crank and crankshaft. The motion of the piston, piston rod and crosshead is to and fro. This to and fro motion is converted into rotary motion with the help of the connecting rod, crank and the crank pin as shown in fig. 9-1. The end of the connecting rod which is attached to the crosshead can only move in a straight line, while the other end attached to the crank pin can move only in a circle, since, the crank carrying the crank pin is free to turn the crankshaft. The motion of the connecting rod is, thus, oscillating. Since, the crank is fixed on the crankshaft, the crankshaft will rotate in its bearing. The flywheel is mounted on the crankshaft.

The following terms are useful in understanding the working of the steam engine:

- **Cylinder bore**: The inside diameter of the cylinder or the liner.
- **Piston stroke**: The distance travelled (or moved) through by the piston from one end of the cylinder to the other end, while the crank is making half a revolution.
Two strokes of the piston are performed per revolution of the crankshaft.

The forward stroke is made by the piston while it moves from the head end to the crank end.

The return stroke is made while the piston travels from the crank end to the head end.

The crank throw or crank radius is the distance between the centre of the crankshaft and the centre of the crank pin. This distance is equal to half the travel of the piston, i.e., piston stroke.

The piston displacement or swept volume is the volume swept by the piston while moving from one end of the cylinder to the other end, while the crank is making half a revolution.

The average linear piston speed is the rate of motion of the piston expressed in metres/minute. It is equivalent to twice the product of the piston stroke (l) in metres and the number of revolutions of the engine per minute (N), i.e., piston speed = 2lN metres/minute.

The dead centres are the positions of the piston at the end of the stroke when the centre lines of the piston rod, the connecting rod and the crank are in the same straight line. There are two such dead centres, one for each end of the piston stroke. For horizontal engines, the two dead centres are known as inner dead centre and outer dead centre. For vertical engines, the two dead centres are known as top dead centre and bottom dead centre.

Clearance volume is the volume of the space between the piston and the cylinder cover, when the piston is at the end of the stroke, plus the volume of the steam port leading to this space.

9.6 Hypothetical Indicator Diagram

An indicator diagram is a plot of steam pressure in the cylinder on the basis of steam volume during the cycle of operations. The theoretical indicator diagram can be constructed geometrically or by calculating pressure of steam at different points of the stroke by applying the law, $pv = \text{constant}$, for the expansive working of steam and then raising the ordinates to get the points of the curve. Such a diagram (fig. 9-5) is known as theoretical or hypothetical indicator diagram because it is constructed after making certain assumptions as follows:

- The opening and closing of the ports is sudden.
- There is no pressure drop due to condensation.
- There is no pressure drop due to wire-drawing which is due to restricted port or valve opening.
- There is no compression to create the cushioning effect on the piston at the end of the stroke.
- The steam is admitted at boiler pressure and exhausted at atmospheric or condenser pressure.
- The expansion of steam is hyperbolic, following the law $pv = \text{constant}$.
- The clearance volume is neglected.

Referring to fig. 9-5, steam is admitted at point a. It is termed as the point of
admission. Steam is supplied to the cylinder at constant pressure, \( p_1 \) up to point of cut-off, \( b \). Steam then expands in the cylinder as the piston moves further. This is accompanied by fall in pressure. The law of expansion \( bc \) is assumed as \( pv = \text{constant} \). Expansion is carried to the end of the stroke. At point \( c \), release of steam takes place. The pressure falls instantaneously to exhaust pressure or back pressure, \( p_b \) represented by line \( cd \). Exhaust takes place at constant pressure \( p_b \) and conditions throughout the whole exhaust stroke are represented by line \( de \).

From a thermodynamic standpoint, it should be assumed that the expansion of an ideal diagram is adiabatic. However, such a condition never occurs in practice because some of the steam condenses before cut-off takes place and the steam is always more or less wet at the beginning of expansion, unless the steam is highly superheated at admission. Condensation also proceeds during part of the expansion and then a certain amount of re-evaporation occurs towards the end of expansion. All these factors take the expansion curve away from an adiabatic curve and very close to \( pv = \text{constant} \) curve or hyperbolic curve.

9.6.1 Hypothetical Mean Effective Pressure: As shown earlier, if the pressure and volume are plotted in kPa and m\(^3\) respectively, the area of \( p-v \) diagram represents the work done in kJ.

(i) Hypothetical indicator diagram without considering clearance and compression:

Referring to fig. 9-6,

Let \( p_1 = \) admission pressure kPa (or kN/m\(^2\)),

\( p_b = \) back pressure in kPa (or kN/m\(^2\)),

\( v_1 = \) volume of steam in the cylinder at cut-off in m\(^3\), and

\( v_2 = \) swept volume in m\(^3\).

Work done per cycle in kJ = area of the hypothetical indicator diagram

\[ = \text{area } a-b-c-d-e \]
\[ = \text{area } a-b-g-o \text{ plus area } b-c-f-g \text{ minus area } d-e-o-f \]
\[ = p_1 v_1 + p_1 v_1 \log_e \left( \frac{v_2}{v_1} \right) - p_b v_2 \]
\[ = p_1 v_1 \left[ 1 + \log_e \left( \frac{v_2}{v_1} \right) \right] - p_b v_2 \]  \( (9.1) \)

Hypothetical m.e.p. = \[ \frac{\text{Area of hypothetical indicator diagram}}{\text{Length of the base of hypothetical indicator diagram}} \]
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\[ \text{Work done per cycle in } \text{kJ} = \frac{p_1 v_1 \left[ 1 + \log_e \left( \frac{v_2}{v_1} \right) \right] - p_b v_2}{v_2} \]

But, \( \frac{v_2}{v_1} = r \) (expansion ratio)

\[ \therefore \text{Hypothetical m.e.p.} = \frac{p_1}{r} \left[ 1 + \log_e (r) \right] - p_b \text{ kPa (or kN/m}^2) \]...
\[ \text{(9.2)} \]

where \( p_1 \) and \( p_b \) are pressures of steam in kPa (kN/m\(^2\)).

It may be noted that \( \frac{v_1}{v_2} = \frac{1}{r} = \text{cut-off ratio} \).

Problem-1: Steam is admitted to the cylinder of a steam engine at a gauge pressure of 1,050 kPa (1.05 MPa) and is cut-off at \( \frac{1}{3} \) stroke. The back pressure is 15 kPa. Calculate the hypothetical mean effective pressure in kPa and kN/m\(^2\) on the piston during the stroke. The barometer reads 96 kPa. Neglect clearance.

Using eqn. (9-2), Hypothetical m.e.p. = \( \frac{p_1}{r} \left[ 1 + \log_e (r) \right] - p_b \) kPa

Here absolute pressure, \( p_1 = \text{gauge pressure} + \text{atmospheric pressure} \)

\[ = 1,050 + 96 = 1,146 \text{ kPa} \]

Expansion ratio, \( r = \frac{1}{\frac{1}{3}} = 3 \), and Back pressure, \( p_b = 15 \text{ kPa} \).

Substituting the values in eqn. (9.2), we have

Hypothetical m.e.p. = \( \frac{1,146}{3} \left[ 1 + \log_e (3) \right] - 15 \)

\[ = \frac{1,146}{3} [1 + 1.0986] - 15 \]

\[ = 786.66 \text{ kPa} = 786.66 \text{ kN/m}^2 \]

Note: 1 kPa = 1 kN/m\(^2\) and 1 MPa = 10 bar = 1 MN/m\(^2\).

Hypothetical indicator diagram considering clearance and compression:

Refer to fig. 9-7. The compression starts at \( e \) and finishes at \( f \) which is also the point of admission. The admission is continued up to \( b \). The expansion curve is represented by \( bc \). \( c \) is the point of release and \( de \) is exhaust line. The compression curve is represented by \( ef \). The expansion and compression curves are assumed to be hyperbolic.

Let \( v \) = volume swept by the piston in one stroke in m\(^3\),
\[ c = \text{fractional ratio of clearance volume to swept volume } v, \]
\[ x = \text{fractional ratio of the volume between points of compression and admission to the swept volume} \ v, \]
\[ k = \frac{1}{r} = \text{fractional ratio of the volume between the points of admission and cut-off to the swept volume} \ v, \]

\[ P_1 = \text{initial pressure of steam or boiler pressure in kPa, and} \]
\[ P_b = \text{back pressure of steam or condenser pressure in kPa.} \]

Net work done per cycle = area \(a-b-c-d-e-f\) minus area \(e-d-w-t\) minus area \(f-e-t-r\)

\[ \text{Hypothetical m.e.p. in kPa} = \frac{\text{Work done per cycle in kJ}}{\text{Swept volume in m}^3} \]

\[ = \frac{1 + r + c}{r + c} \log_e \left( \frac{1 + c}{r + c} \right) - p_b \left[ (1 - x) + (x + c) \log_e \left( \frac{x + c}{c} \right) \right] \]

\[ (9.3) \]

\[ \text{Hypothetical m.e.p.} \]

\[ = p_1 \left[ \frac{1}{r + c} \log_e \left( \frac{1 + c}{1 + c} \right) \right] - p_b \left[ (1 - x) + (x + c) \log_e \left( \frac{x + c}{c} \right) \right] \]

\[ (9.4) \]

\[ \text{If the compression effect is neglected, i.e.} \ x = 0, \ \text{the eqn. (9.4) becomes,} \]

\[ \text{Hypothetical m.e.p.} = p_1 \left[ 1 + c \log_e \left( \frac{c}{c} \right) \right] - p_b \left[ 1 + c \log_e \left( \frac{c}{c} \right) \right] \]

\[ (9.5) \]

\[ \text{If the effect of clearance volume is neglected, i.e.,} \ c = 0, \ \text{the eqn. (9.5) becomes,} \]
Hypothetical m.e.p. = \( \frac{p_1}{r} \left[ 1 + \log_e \left( \frac{1 + c}{r + c} \right) \right] - p_b \) 

\[ \text{...(9.6)} \]

This is same as expression (9.2) directly derived earlier.

Point of cut-off is the point at which the supply of steam to the cylinder is closed. Cut-off ratio is expressed as the ratio of the volume of steam in the cylinder when the piston is at the point of cut-off, and the volume of steam in the cylinder when the piston is at the end of the stroke. If the clearance is not considered while taking the volume of steam at the point of cut-off and at the end of the stroke, the cut-off is known as apparent cut-off. If the clearance is taken into account, this ratio is known as real cut-off. Thus, referring to fig. 9-7,

\[ \text{Apparent cut-off ratio} = \frac{kv}{v} = k = \frac{1}{r} \]  
\[ \text{Real cut-off ratio} = \frac{kv + cv}{v + cv} = \frac{k + c}{1 + c} \]  

\[ \text{...(9.7)} \]
\[ \text{...(9.8)} \]

Ratio of expansion is the ratio of the volume of steam at the end of the stroke, and the volume of steam at the point of cut-off. It is the reciprocal of the cut-off ratio. If no account is taken of the clearance volume while considering this ratio, the ratio is called apparent or normal ratio of expansion. If the clearance volume is considered, the ratio is known as real or actual ratio of expansion.

\[ \text{Apparent expansion ratio} = \frac{v}{kv} = \frac{1}{k} = r \]  
\[ \text{and real expansion ratio} = \frac{v + cv}{kv + cv} = \frac{1 + c}{k + c} \]  

\[ \text{...(9.9)} \]
\[ \text{...(9.10)} \]

**Problem-2** : Steam is admitted to the cylinder of a single cylinder engine at a pressure of 1,400 kPa and it cut-off at 0.25 of the stroke. The back pressure is 120 kPa and the clearance volume is 10% of the swept volume. Calculate the hypothetical (theoretical) mean effective pressure on the piston during the stroke.

Using eqn. (9.5),

\[ \text{Hypothetical m.e.p.} = p_1 \left[ \frac{1}{r} + \left( \frac{1}{r} + c \right) \log_e \left( \frac{1 + c}{r + c} \right) \right] - p_b \]

Here, \( \frac{1}{r} = 0.25 \), \( c = 0.1 \), \( p_1 = 1,400 \) kPa, and \( p_b = 120 \) kPa.

\[ \therefore \text{Hypothetical m.e.p.} = 1,400 \left[ 0.25 + (0.25 + 0.1) \log_e \left( \frac{1 + 0.1}{0.25 + 0.1} \right) \right] - 120 \]
\[ = 1,400 \left[ 0.25 + 0.35 \times 1.1442 \right] - 120 = 790.66 \text{ kPa} \]

**Problem-3** : The cylinder of a non-condensing steam engine is supplied with steam at 1,220 kPa. The clearance volume is 1/10th of the stroke volume and the cut-off takes place at 1/4th of the stroke. If the pressure at the end of compression is 550 kPa, calculate the value of mean effective pressure of the steam on the piston. Assume that expansion and compression are hyperbolic. Take back pressure as 110 kPa.

Let \( v = \) stroke volume, and \( cv = \) clearance volume,

then the volume of steam at the beginning of the compression stroke = \( v (x + c) \)
where, \( x \) = compression ratio without considering clearance volume, and the volume of steam at the end of compression = \( cv \).

Applying hyperbolic law between the points of beginning and end of compression,

\[
110 (c + x) v = 550 \times cv
\]

i.e., \( 110 (0.1 + x) v = 550 \times 0.1 v \) i.e., \( 0.1 + x = 0.5 \) \( \therefore x = 0.4 \)

Here, \( \frac{1}{r} = 4, c = 0.1, p_t = 1,220 \text{kPa}, p_b = 110 \text{kPa}, \) and \( x = 0.4 \).

Now, using eqn. (9.4). Hypothetical m.e.p.

\[
= p_t \left[ 1 + \frac{1 + c}{1 + r} \log_e \left( \frac{1 + c}{1 + r} \right) \right] - p_b \left[ (1 - x) + (x + c) \log_e \left( \frac{x + c}{c} \right) \right]
\]

\[
= 1,220 \left[ 1 + \frac{1 + 0.1}{1 + 0.4} \log_e \left( \frac{1 + 0.1}{1 + 0.4} \right) \right] - 110 \left[ (1 - 0.4) + (0.4 + 0.1) \log_e \left( \frac{0.4 + 0.1}{0.1} \right) \right]
\]

\[
= 1,220 [0.25 + 0.35 \log_e 3.14] - 110 [0.6 + 0.5 \times \log_e 5]
\]

\[
= 1,220 [0.25 + 0.35 \times 1.1442] - 110 [0.6 + 0.5 \times 1.6094] = 639.1 \text{kPa}
\]

**Problem-4**: In a steam engine, steam is admitted for 40% of the working stroke. Clearance volume is 10% of the swept volume. Calculate the apparent and real values of cut-off and expansion ratios.

Assuming the swept volume = 100 units,

Apparent ratio of cut-off = \( \frac{40}{100} = 0.4 \) and

Apparent ratio of expansion = \( \frac{100}{40} = 2.5 \)

Real ratio of cut-off = \( \frac{\text{total volume of steam at cut-off}}{\text{total cylinder volume}} \)

\[
= \frac{\text{total volume of steam at cut-off}}{\text{swept volume + clearance volume}} = \frac{40 + 10}{100 + 10} = 0.454
\]

Real ratio of expansion = \( \frac{\text{total cylinder volume}}{\text{total volume of steam at cut-off}} = \frac{100 + 10}{40 + 10} = 2.2 \)

### 9.7 Cylinder Condensation

If saturated steam is brought in contact with a surface colder than temperature of the steam, condensation of steam begins immediately. During condensation the steam gives up part of its latent heat and becomes wetter.

During the period of exhaust, the cylinder walls are cooled by contact with the relatively cool low pressure exhaust steam. When the hot steam from boiler is admitted to the cylinder, a part of it condenses as it comes in contact with previously cooled cylinder walls during the exhaust stroke. The water thus formed by the process of condensation is deposited in the form of a thin film on the walls of the cylinder. The heat received by the cylinder walls from the fresh hot steam will be given back to the deposited film of water during the time the steam expands and consequently falls in temperature after the cut-off takes place. This transferred heat from the cylinder walls
to the water film will re-evaporate some of the water into steam at the end of expansion stroke.

The condensation, thus, takes place during the early part of the admission stroke, re-evaporation occurs partly towards the end of the expansion stroke, and partly during exhaust stroke. The re-evaporation during expansion period behind the piston increases the total work done. But, the re-evaporated steam during the exhaust period in a single cylinder engine passes away to waste, without doing any useful work. However, in a compound steam engine this re-evaporated steam will be utilized in the next cylinder to which the steam is led from the first cylinder. Any degree of condensation results in loss of energy available for doing useful work, and this loss may vary from 10% to 40% of the available energy of the steam.

The methods adopted to reduce the amount of condensation of steam in the cylinder are:

- Obtaining the steam from the boiler in superheated state and lagging the steam pipe from boiler to engine by non-conducting material such as asbestos.
- Jacketing the cylinder with hot steam from the boiler.
- Compounding the cylinders, and thereby reducing the range of variation of temperature in each cylinder.

9.8 Actual Indicator Diagram

An actual indicator diagram obtained on a steam engine is more likely to be of the form shown dotted in fig. 9-8. The hypothetical indicator diagram of this engine is shown in full lines.

The difference between two diagrams is on account of certain practical factors given below, which were not considered in arriving at the hypothetical indicator diagram:

- The big drop in the steam pressure between boiler and the engine cylinder is due to condensation caused by loss of heat in steam pipes, friction losses in the steam supply pipe and admission valve and to wire-drawing in the valves. This pressure difference will tend to increase toward the point of cut-off due to increasing velocity of the piston and consequent increased demand for steam and due to condensation of steam in the cylinder.

.. The opening and closing of the ports is a gradual process as the valve moves over it. The pressure changes at the opening and closing of ports are not as sudden as shown on the hypothetical diagram, and there is rounding-off of the actual diagram at the beginning of admission, cut-off and release.

.. The actual expansion curve is not a true hyperbola owing to the varying intercharge of heat through the cylinder walls. At the commencement of the expansion, the steam in
the cylinder is hotter than the cylinder wall; this causes condensation of steam and the volume of steam consequently decreases. Near the end of the expansion stroke, owing to the low pressure of steam, the steam is colder than the cylinder walls; this causes heat to flow from the cylinder walls to the steam which tends to re-dry the steam.

The release (opening of exhaust port) occurs before the end of the expansion stroke, and the time factor in the opening of the exhaust port will cause the rounding-off of the toe of the actual diagram.

Exhaust pressure in the cylinder will not be quite so low as the condenser pressure because the size of the exhaust port is limited and therefore, the exhaust from the port is not quick.

There is rounding-off of the heel of the actual diagram due to closing of the exhaust port before the end of the stroke. This results in compression of the steam in the cylinder, which serves as a soft cushion for bringing the piston to rest at the end of the stroke. Admission occurs just before the end of the compression stroke.

It will be seen from fig. 9-8 that the area of the actual indicator diagram is less than the area of the hypothetical indicator diagram. The ratio between the areas of these two diagrams is known as the diagram factor. Or

\[ f = \frac{\text{Area of actual indicator diagram}}{\text{Area of hypothetical indicator diagram}} = \frac{\text{mean height of actual indicator diagram}}{\text{mean height of hypothetical indicator diagram}} = \frac{\text{m.e.p. from actual indicator diagram}}{\text{m.e.p. from hypothetical indicator diagram}} \]

The diagram factor varies slightly with different types of engines, and its average value is about 0.75.

9.8.1 Indicator: Instrument used for drawing diagrams showing actual pressure–volume relations within the engine cylinder during one revolution of the engine crank is called indicator. The diagram is drawn on a piece of paper usually called an indicator card. The diagram showing actual pressure–volume relations within the engine cylinder is called an indicator diagram (fig. 9-8). The length of the indicator diagram represents the length of the stroke to a reduced scale and its height at any point represents the pressure on the piston at the corresponding point in the stroke. For double-acting steam engines a separate diagram is taken for each end of the cylinder.

A common form of steam engine indicator, shown in fig. 9-9, consists of two main parts:
- a small cylinder, communicating at one end with the engine cylinder, and
- a drum for holding the indicator card.

The small cylinder contains a closely fitted piston to which is attached a strong
helical spring. The piston is forced upward against the resistance of this spring by the steam pressure in the engine cylinder and forced downward by the spring when the steam pressure drops. The end of the piston rod of the indicator piston is attached through a series of links to a pencil lever that has a pencil point on its free end. The links are so designed that they magnify the motion of the indicator piston at the pencil point and yet cause it to move in a truly vertical line.

A coiled spring attached to the inside of the drum resists the pull of the cord on the outward stroke and keeps the cord tight on the return stroke.

In operation, the indicator card is wrapped around the indicator drum and held against it by spring clamps. One end of a fine cord is attached to and wrapped several times around the groove at the base of the drum, and the other end is attached to some form of reducing motion, which in turn is connected to a reciprocating part of the engine, preferably to the engine crosshead pin. When indicator cylinder is put in communication with the engine cylinder, the lever carrying the pencil point will rise and fall according to the rise and fall of steam pressure. At the same time the drum will be rotated back and forth in some proportion to the engine stroke. Now, if the pencil point is pressed against the indicator card, it will trace an indicator diagram on the indicator card.

9.9. Power and Efficiencies

9.9.1 Indicated Power: The power developed in the cylinder of any engine is commonly known as indicated power. Thus, indicated power is the rate of doing work on the moving piston in the cylinder. It is called indicated power because an instrument known as the indicator, is used to measure it. The power developed in the cylinder of any engine can be determined, if the engine cylinder diameter, piston stroke and speed are known, and if an indicator diagram with its spring scale or number is available. The first step is to find the mean effective pressure. The mean effective pressure is the average effective pressure on the piston.

\[
\text{Mean height of indicator diagram in mm, } h = \frac{\text{area of indicator diagram in } \text{mm}^2}{\text{length of indicator diagram in } \text{mm}}
\]

Indicated mean effective pressure in kPa or kN/m² = mean height, \( h \) in mm x spring scale or spring number, kPa or kN/m² per 1 mm elongation.

The mean effective pressure is a constant pressure which will do the same work as the actual varying pressure. Hence, the actual varying pressure in the engine cylinder can be replaced by the mean effective pressure. To find the power, we must know the work done per second. This will be equal to the amount of work done per stroke multiplied by number of working strokes per second.

If \( p_m \) = actual mean effective pressure of steam in N/m² or Pa,

\[
a = \text{area of the piston in } \text{m}^2,
\]

\[
l = \text{length of piston stroke in metre, and}
\]

\[
N = \text{number of revolutions made by the engine per second (r.p.s)},
\]

Then, driving force on the piston during a stroke = \( p_m \times a \) newtons

and Work done per stroke = \( p_m \times a \times l \) N-m (newton-metre) or J (joules)

Since two working strokes are obtained in one revolution in a double-acting steam engine,
Work done per second = $p_m \times a \times I \times 2N \, J/s \text{ or } W \, (\text{watts})$

:. Indicated power = $2 \times p_m \times a \times I \times N \, W$

or = $\frac{2 \times p_m \times a \times I \times N}{1,000} \, \text{kW}$ \hspace{1cm} \ldots(9.12)

A portion of the power developed in the engine cylinder is absorbed in overcoming the friction of the moving parts of the engine itself. The remainder is available for doing the required work. The power absorbed in overcoming the frictional resistance of the moving parts of the engine is termed frictional power.

9.9.2. Brake Power: The actual power available from the engine for doing useful work is termed the brake power or shaft power. The brake power of an engine can be determined by a brake of some kind applied to the brake pulley of the engine. The arrangement for the determination of brake power of the engine is known as dynamometer.

Dynamometers are broadly divided into two classes:

- Absorption dynamometers, and
- Transmission dynamometers.

In absorption dynamometer, the power available from the engine is absorbed in the form of friction at the brake. In transmission dynamometer, the power available from the engine is not wasted in friction. Transmission dynamometer transmits the power and measures it at the same time.

Absorption brake dynamometers are those that absorb the power to be measured by friction. This power absorbed in friction is finally dissipated in the form of heat energy. Common forms of absorption dynamometers are: the rope brake, the prony brake, the fan brake, the hydraulic brake and the electrical brake.

Rope brake dynamometer is a convenient type of brake and can be applied to the flywheel of a moderate size engine. The rope brake consists of a double rope passed round the brake wheel as shown in fig. 9-10, the upper end is connected to a spring balance suspended from overhead and the lower end carries the load $W$ in newtons. The load $W$ is usually called dead load. The ropes are generally held apart by the wooden blocks.

![Fig 9-10 Rope brake dynamometer.](image)

The pull $S$ in newtons indicated by the spring balance, is helping to turn the wheel while the load $W$ is opposing its rotation. The direction of the rotation of the wheel is shown by the arrow. At the given load the whole power developed by the engine is absorbed by the friction produced at the rim of the wheel. Since, the whole power is converted into heat, the rim of the wheel must be water cooled.
The net resistance against the wheel or the frictional force or the net load = \( (W - S) \) newtons. If \( R \) is the effective radius of the wheel in metres which works as the torque arm for frictional resistance, i.e. \( R = \frac{D + d}{2} \) (wheel \( D = \) diameter of the brake wheel and \( d = \) diameter of the brake rope) then, the frictional torque = the resisting force \( \times \) arm = \( (W - S) \times R \) N.m.

If \( N = \) number of revolutions made by the engine per second (r.p.s), then the number of radians per second = \( 2\pi N \).

The work absorbed = \( (W - S)R \times 2\pi N \) N.m/s or J/s or W (watts)

\[
\text{:. Brake power} = \frac{(W-S) R \times 2\pi N}{1,000} \text{ kW}
\]

\[
\begin{align*}
\text{:. Brake power} &= \frac{\text{Torque} \times 2\pi N}{1,000} \text{ kW}
\end{align*}
\]

Power lost in overcoming the friction of rotating and sliding parts of the engine is the difference between the indicated power and brake power.

\[
\text{i.e. Friction power} = \text{Indicated power} - \text{Brake power} \quad (9.14)
\]

9.9.3 Mechanical, Thermal and Overall Efficiencies: The performance of steam engines is usually stated in terms of the followings:

- Mechanical efficiency,
- Thermal efficiency, and
- Overall efficiency.

The mechanical efficiency of engine takes into account the energy loss due to the friction of the moving parts of the engine, and is calculated from the relation between the brake power and the indicated power. The power lost in friction is the difference between the indicated power and the brake power.

It is generally assumed that friction power is the same at all loads at constant speed. However, various tests indicate that the friction power generally increase with the load but not to any considerable extent.

The mechanical efficiency is defined as the ratio of the power output of the engine, as measured by the brake, to the power developed by the steam in the engine cylinder, as obtained from the indicator diagram,

\[
\text{i.e. Mechanical efficiency, } \eta_m = \frac{\text{Brake power}}{\text{Indicated power}} \quad (9.15)
\]

The mechanical efficiency of high speed steam engines at full load is generally between 80 to 90 per cent. With forced lubrication, its value at full load is still higher.

The thermal efficiency is defined as the ratio of the heat converted into useful work, to the heat supplied. The heat supplied per kg of steam to the engine may be worked out as the difference between the enthalpy, \( H_1 \) of steam at conditions existing at engine stop valve and the enthalpy \( h_2 \) of water at the temperature of engine exhaust (heat of water or condensate returned to hot-well). So, heat (net) supplied in kJ per kg of steam = \( H_1 - h_2 \).

The useful work obtained may be worked out either from indicated power or brake power. If the heat equivalent of indicated power developed by the engine is considered
as the output, the efficiency is termed *indicated thermal efficiency*,

\[
\text{i.e. Indicated thermal efficiency, } \eta_i = \frac{\text{Heat equivalent of indicated power in kJ/sec}}{\text{Heat supplied in steam in kJ/sec}} = \frac{\text{Indicated power in kW}}{m_s (H_1 - h_2)} \quad \text{(9.16)}
\]

where, \( m_s \) = steam consumption in kg/sec,

\( H_1 \) = enthalpy of 1 kg of steam supplied in kJ, and

\( h_2 \) = enthalpy of 1 kg of water of exhaust steam in kJ.

Brake thermal efficiency, \( \eta_b = \frac{\text{Heat equivalent of brake power in kJ/sec}}{\text{Heat supplied in steam in kJ/sec}} = \frac{\text{Brake power in kW}}{m_s (H_1 - h_2)} \quad \text{(9.17)}
\]

where, \( m_s \) = steam consumption in kg/sec,

\( H_1 \) = enthalpy of 1 kg of steam supplied in kJ, and

\( h_2 \) = enthalpy of 1 kg of water of exhaust steam in kJ.

Comparing eqns. (9.15), (9.16) and (9.17),

Mechanical efficiency, \( \eta_m = \frac{\text{Brake thermal efficiency}}{\text{Indicated thermal efficiency}} \quad \text{(9.18)}
\]

The overall efficiency of *steam engine plant* is defined as the ratio of the power output of the engine as measured by the brake to the heat energy supplied by the fuel,

\[
\text{i.e. Overall efficiency of steam engine plant} = \frac{\text{Heat equivalent of brake power in kJ/sec}}{\text{Heat supplied by fuel in kJ/sec.}} = \frac{\text{brake power in kW}}{m_f \times C.V.} \quad \text{(9.19)}
\]

where, \( m_f \) = fuel supplied to the boiler in kg/sec, and

\( C.V. \) = calorific value of fuel used in the boiler in kJ/kg.

**Problem-5:** A single-cylinder, double-acting steam engine of 20 cm diameter and 40 cm stroke is supplied with steam at 834 kPa and exhausts at 147 kPa. Cut-off takes place at \( \frac{1}{3} \)rd stroke and the engine runs at 2 r.p.s. Using a diagram factor of 0.7, estimate the actual mean effective pressure and the indicated power of the engine.

Here, \( p_1 = 834 \text{ kPa} \), Expansion ratio, \( r = \frac{1}{\text{cut-off}} = \frac{1}{\frac{1}{3}} = 3 \), and \( p_b = 14.7 \text{ kPa} \).

Using eqn. (9.2), hypothetical m.e.p.

\[
= \frac{p_1}{r [1 + \log_e (r)] - p_b} = \frac{834}{3 [1 + \log_e (3)] - 14.7} = 568.6 \text{ kPa}
\]

Actual m.e.p., \( p_m = \text{diagram factor, } f \times \text{theoretical m.e.p.} = 0.7 \times 568.6 = 398.02 \text{ kPa} \).

Using eqn. (9.12), Indicated power = \( 2 \times p_m \times a \times l \times N \) watts
where, \( p_m \) = actual m.e.p. = \( 398.02 \times 10^3 \) Pa

\[
a = \text{area of the piston} = \pi \left( \frac{20}{100} \right)^2 \text{m}^2,
\]

\[
l = \text{length of piston stroke} = \frac{40}{100} \text{ m}, \text{ and}
\]

\[
N = \text{number of revolutions made by the engine} = 2 \text{ r.p.s.}
\]

Substituting the above values,

\[
\text{Indicated power} = 2 \times 398.92 \times \frac{10^3}{4} \left( \frac{20}{100} \right)^2 \times \frac{40}{100} \times 2
\]

\[= 20,000 \text{ W or } 20 \text{ kW}.
\]

**Problem-6:** Find the dimensions of a simple steam engine cylinder to develop indicated power of 50 kW. The steam supply is at 800 kPa. The engine makes 2 r.p.s. and is double-acting. Cut-off is at \( \frac{3}{8} \) of the stroke, diagram factor is 0.65 and the back pressure is 120 kPa. The stroke is 1.5 times the diameter of the cylinder, and the clearance volume is 8% of the stroke volume.

Using eqn. (9.5), Hypothetical m.e.p. = \( p_t \left[ \frac{1}{r} + \left( \frac{1}{r} + c \right) \log_e \left( \frac{1+c}{1} \right) \right] - p_b \)

Here, \( c = \frac{\text{clearance volume}}{\text{stroke volume}} = \frac{8}{100} = 0.08 \) (assuming stroke volume as 100 units),

Expansion, ratio, \( r = \frac{1}{\text{cut-off}} = \frac{1}{\frac{3}{8}} = \frac{8}{3} = 2.666, p_t = 800 \text{ kPa and } p_b = 120 \text{ kPa}.

Hypothetical m.e.p. = \( 800 \left[ \frac{1}{2.666} + \left( \frac{1}{2.666} + 0.08 \right) \log_e \left( \frac{1+0.08}{1} \right) \right] - 120 \)

\[= 800 \left[ 0.375 + 0.455 \times 0.8642 \right] - 120 = 495 \text{ kPa}
\]

:. Actual m.e.p., \( p_m = \text{hypothetical m.e.p.} \times f = 495 \times 0.65 = 322 \text{ kPa} \)

Let cylinder diameter = \( d \) metre, then piston stroke, \( l = 1.5d \) metre.

Indicated power = \( 2 \times p_m \times a \times l \times N \text{ watts} \)

[where \( p_m \) is indicated m.e.p. in \( p_d \) (pascals)]

i.e. \( 50 \times 10^3 = 2 \times 322 \times 10^3 \times \pi \frac{d^2}{4} \times 1.5d \times 2 \)

\[= 0.03295 \]

:. Diameter of the cylinder = \( 0.03295 = 0.32 \text{ m or } 32 \text{ cm} \).

and Piston stroke, \( l = 1.5d = 1.5 \times 32 = 48 \text{ cm} \).

**Problem-7:** The diameter of the cylinder of a simple, double-acting steam engine is 30 cm. The stroke is 38 cm. The steam is admitted to the cylinder at a pressure of 700 kPa and is cut-off when the piston has advanced 9 cm from the dead centre position. Assume a diagram factor of 0.7, calculate the indicated power of the engine running at 150 r.p.m. Assume back pressure of 110 kPa. Neglect clearance effect.
Here, Cut-off = 9/38 = 0.237 and expansion ratio, \( r = 1/0.237 = 4.22 \),
\( p_f = 700 \text{ kPa} \), and \( p_b = 110 \text{ kPa} \).

Hypothetical m.e.p. = \( \frac{p_f}{r} [1 + \log_e (r)] - p_b \)
\[ = \frac{700}{4.22} [1 + \log_e (4.22)] - 110 = 294 \text{ kPa} \]

Actual m.e.p., \( p_m = \) hypothetical m.e.p. \( \times f = 294 \times 0.7 = 206 \text{ kPa} \)

Indicated power = \( 2 \times p_m \times a \times l \times N \text{ watts} \)
\[ = 2 \times 206 \times 10^3 \times \frac{\pi}{4} \times (0.3)^2 \times \frac{38}{100} \times \frac{150}{60} \]
\[ = 27,670 \text{ watt or } 27.67 \text{ kW} \]

Problem-8: The following data relate to a test on a double-acting, single-cylinder steam engine:

Indicated power, 80 kW; engine speed, 140 r.p.m.; cylinder diameter, 30 cm; piston stroke, 45 cm; steam pressure at admission, 1,100 kPa; cut-off at \( 1/3 \)rd stroke and back pressure, 40 kPa. Neglecting the effect of clearance volume, calculate the diagram factor.

Here, expansion ratio, \( r = 1/1/3 = 3 \), \( p_f = 1,100 \text{ kPa} \), and \( p_b = 40 \text{ kPa} \).

Using eqn. (9.2), Hypothetical m.e.p. = \( \frac{p_f}{r} [1 + \log_e (r)] - p_b \)
\[ = \frac{1,100}{3} [1 + \log_e (3)] - 40 = 729.48 \text{ kPa} \]

Indicated power = \( 2 \times p_m \times a \times l \times N \text{ W} \)
i.e. \( 80 \times 10^3 = 2 \times p_m \times \frac{\pi}{4} \times (0.3)^2 \times 0.45 \times \frac{140}{60} \)

Actual m.e.p., \( p_m = \frac{80 \times 10^3 \times 4 \times 60}{2 \times 3.14 \times (0.3)^2 \times 0.45 \times 140} = 539.21 \text{ Pa or } 539.21 \text{ kPa} \)

Diagram factor, \( f = \frac{\text{Actual m.e.p. in kPa}}{\text{Hypothetical m.e.p. in kPa}} = \frac{539.21}{729.48} = 0.739 \)

Problem-9: A double-acting, steam engine receives steam at a pressure of 7 bar. The cut-off takes place at \( 1/4 \)th stroke and the diagram factor is 0.7. The indicated power of the engine is 50 kW and the engine runs at 3 revolutions per second. Find the dimensions of the steam engine if the piston stroke is 1.5 times the diameter of the cylinder. The back pressure is 1.1 bar. Neglect clearance effect.

Here, Expansion ratio, \( r = \frac{1}{\text{cut-off}} = \frac{1}{1/4} = 4 \), \( p_f = 7 \text{ bar} \), \( p_b = 1.1 \text{ bar} \) and \( f = 0.7 \).

Using eqn. (9.2), Hypothetical m.e.p. = \( \frac{p_f}{r} [1 + \log_e (r)] - p_b \)
\[ = \frac{7}{4} [1 + \log_e (4)] - 1.1 = 3.08 \text{ bar} \]

Actual m.e.p., \( p_m = \) f \times theoretical m.e.p. = 0.7 \times 3.08 = 2.156 \text{ bar}
Indicated power = $2 \times p_m \times a \times l \times N \ W$

i.e. $50 \times 10^3 = 2 \times (2.156 \times 10^5) \times \frac{\pi}{4} d^2 \times 1.5d \times 3$

\[d^3 = \frac{50 \times 10^3 \times 4}{2 \times 2.156 \times 10^5 \times 3.14 \times 1.5 \times 3} = 0.0328\]

\[d = \sqrt[3]{0.0328} = 0.32 \text{ m, or } 32 \text{ cm}\]

\[\text{Piston stroke, } l = 1.5d = 1.5 \times 32 = 48 \text{ cm}\]

Problem-10: A simple, double-acting steam engine having a cylinder diameter 35 cm and stroke 53 cm takes steam at a pressure of 800 kPa and exhaust takes place at 100 kPa. Find the increase in power when cut-off is changed from 0.4 to 0.5 of the stroke. The engine speed is 200 r.p.m. and the diagram factor can be taken as 0.85 in both the cases. Neglect clearance effect.

(i) Here, Expansion ratio, $r = \frac{1}{0.4} = 2.5$, $f = 0.85$, $P_1 = 800 \text{ kPa}$ and $P_b = 100 \text{ kPa}$

Using eqn. (9.2), Hypothetical m.e.p. = $\frac{P_1}{r} [1 + \log_e (r)] - P_b = \frac{800}{2.5} [1 + \log_e (2.5)] - 100 = 513 \text{ kPa}$

Actual m.e.p., $p_m = \text{hypothetical m.e.p.} \times f = 513 \times 0.85 = 436 \text{ kPa}$

Indicated power = $2 \times p_m \times a \times l \times N \ W$

\[= 2 \times 436 \times 10^3 \times \frac{\pi}{4} (0.35)^2 \times \frac{53}{100} \times \frac{200}{60} = 1,48,200 \text{ W or } 148.2 \text{ kW}\]

(ii) Expansion ratio, $r = \frac{1}{0.5} = 2$, $P_1 = 800 \text{ kPa}$, $P_b = 100 \text{ kPa}$, $f = 0.85$

Hypothetical m.e.p. = $\frac{P_1}{r} [1 + \log_e (r)] - P_b = \frac{800}{2} [1 + \log_e (2)] - 100 = 578 \text{ kPa}$

Actual m.e.p., $p_m = 578 \times 0.85 = 491 \text{ kPa}$

Indicated power = $2 \times p_m \times a \times l \times N \ W$

\[= 2 \times 491 \times 10^3 \times \frac{\pi}{4} (0.35)^2 \times \frac{53}{100} \times \frac{200}{60} = 1,66,900 \text{ W or } 166.9 \text{ kW}\]

Increase in power when cut-off is changed from 0.4 to 0.5

\[= 166.9 - 148.2 = 18.7 \text{ kW}\]

Problem-11: Calculate the theoretical steam consumption in kg per kW per hour of a single-cylinder, double-acting steam engine from the following data:

Diameter of cylinder. 25 cm; stroke, 45 cm; r.p.s., 3; steam pressure, 1,100 kPa; dryness fraction of steam supplied, 0.95; back pressure, 110 kPa; and cut-off takes place at 1/3rd stroke for both sides. Assume a diagram factor of 0.8 and neglect clearance.

Here, Expansion ratio, $r = \frac{1}{\text{cut-off}} = \frac{1}{\frac{1}{3}} = 3$, $P_1 = 1,100 \text{ kPa}$, and $P_b = 110 \text{ kPa}$.
Hypothetical m.e.p. = \( \frac{p_1}{r} [1 + \log_e(r)] - p_b \)

\[ \frac{1,100}{3} [1 + \log_e 3] - 110 = 660 \text{kPa}. \]

Actual m.e.p., \( p_m = 660 \times 0.8 = 528 \text{kPa} \)

Indicated power = \( 2 \times p_m \times a \times l \times N \text{ W} \)

\[ 2 \times 528 \times 10^3 \times \frac{\pi}{4} (0.25)^2 \times \frac{45}{100} \times 3 = 70,000 \text{ W or } 70 \text{ kW} \]

Volume of cylinder = \( \frac{\pi}{4} d^2 \times l = \frac{\pi}{4} \left( \frac{25}{100} \right)^2 \times \frac{45}{100} = 0.0221 \text{ m}^3. \)

Since cut-off takes place at 1/3rd stroke,

Volume of steam admitted per stroke = \( \frac{0.0221}{3} = 0.00737 \text{ m}^3. \)

Since, the engine is double-acting,

Volume of steam admitted per revolution = \( 0.00737 \times 2 = 0.01474 \text{ m}^3. \)

\[ \therefore \text{Volume of steam admitted per hour} = 0.01474 \times 3 \times 3,600 = 159 \text{ m}^3. \]

From steam tables at 1,100 kPa (11 bar), \( v_s = 0.1775 \text{ m}^3/\text{kg}. \)

\[ \therefore \text{Volume of 1 kg of wet steam admitted} = x v_s = 0.95 \times 0.1775 = 0.1686 \text{ m}^3. \]

\[ \therefore \text{Theoretical steam consumption in kg per hour} = \frac{159}{0.1686} = 943 \text{ kg.} \]

\[ \therefore \text{Theoretical steam consumption in kg per kW per hour} = \frac{943}{70} = 13.47 \text{ kg/kW-hr.} \]

Problem-12: The following data were obtained during the trial of single-cylinder, double-acting steam engine:

- Cylinder diameter, 28 cm; stroke, 40 cm; speed, 4 r.p.s.; area of the indicator diagram, 10 cm²; length of indicator diagram, 7.6 cm; scale of indicator spring, 200 kPa/cm; dead load on the brake, 2,119 newtons; spring balance reading, 157 newtons; circumference of brake wheel, 5 m; circumference of brake rope, 8 cm; steam supplied per hour, 435 kg.

Determine the indicated power, brake power, mechanical efficiency, steam consumption per kW per hour on indicated power basis, steam consumption per kW per hour on brake power basis, and power lost in friction.

Actual m.e.p., \( p_m = \frac{\text{area of the indicator diagram}}{\text{length of the indicator diagram}} \times \text{spring scale} \)

\[ = \frac{10}{7.6} \times 200 = 263 \text{kPa} \]

Indicated power = \( 2 \times p_m \times a \times l \times N \text{ W} \)
\[ 2 \times 263 \times 10^3 \times \frac{\pi}{4} (0.28)^2 \times \frac{40}{100} \times 4 = 51,820 \text{ W or } 51.82 \text{ kW} \]

Dia. of brake wheel \( \frac{5 \times 100}{\pi} = 159 \text{ cm} \), and Dia. of brake rope \( \frac{8}{\pi} = 2.54 \text{ cm} \).

Effective radius of brake wheel, \( R = \frac{159 + 2.54}{2 \times 100} = 0.8077 \text{ m} \).

Using eqn. (9.13), Brake Power = \( \frac{(W - S) \times 2\pi \times R \times N}{1,000} \)

\[ = \frac{(2,119 - 157) \times 2\pi \times 0.8077 \times 4}{1,000} = 39.82 \text{ kW} \]

Using eqn. (9.15), Mechanical efficiency, \( \eta_m = \frac{\text{Brake Power}}{\text{Indicated Power}} \)

\[ = \frac{39.82}{51.82} = 0.768 \text{ or } 76.8\% \]

Steam consumption per kW per hour on Indicated power basis = \( \frac{435}{51.82} = 8.394 \text{ kg} \)

Steam consumption per kW per hour on Brake power basis = \( \frac{435}{39.82} = 10.924 \text{ kg} \)

Using eqn. (9.14), Power lost in friction

\[ = \text{Indicated Power} - \text{Brake Power} \]

\[ = 51.82 - 39.82 = 12 \text{ kW} \]

Problem-13: The following data were obtained during the trial of a single-cylinder, double-acting steam engine:

- Speed: 104.5 r.p.m
- Cylinder diameter: 22 cm
- Piston stroke: 30 cm
- M.E.P. (each end): 120 kPa
- Effective brake radius: 60 cm
- Dead load on the brake: 726 newtons
- Spring balance reading: 176 newtons
- Pressure of steam supplied: 700 kPa (7 bar)
- Steam used per hour: 57 kg
- Condensate temperature: 50°C

Steam supplied was dry and saturated. Calculate the mechanical efficiency, and the indicated and brake thermal efficiencies of the engine.

indicated power = \( 2 \times p_m \times a \times I \times N \text{ watts} \)

\[ = 2 \times 120 \times 10^3 \times \frac{\pi}{4} (0.22)^2 \times \frac{30}{100} \times \frac{104.5}{60} \]

\[ = 4,767 \text{ W or } 4.767 \text{ kW} \]
Brake Power = \( \frac{(W - S) R \times 2\pi \times N}{1,000} \) kW

\[
\begin{align*}
(726 - 176) \times \frac{60}{100} \times \frac{2\pi \times 104.5}{60} \times \frac{1,000}{1,000} &= 3.607 \text{ kW}
\end{align*}
\]

Mechanical efficiency, \( \eta_m = \frac{\text{Brake Power}}{\text{Indicated Power}} = \frac{3.607}{4.767} = 0.7567 \) or 75.67%.

At 7 bar (from steam tables) \( H_1 = 2,763.5 \text{ kJ/kg} \).

Heat remaining in condensate, \( h_2 = 4.187 \times (50 - 0) = 209.3 \text{ kJ/kg} \).

Steam used per second, \( m_s = \frac{57}{3,600} = 0.0158 \text{ kg/sec} \).

:. Heat supplied in steam in kJ per second = \( m_s (H_1 - h_2) \text{ kJ/sec} = 0.0158(2,763.5 - 209.3) = 40.36 \text{ kJ/sec} \).

Heat equivalent of indicated power = 4.767 kJ/sec.

Using eqn. (9.16), Indicated thermal efficiency,

\[
\eta_i = \frac{\text{Heat equivalent of indicated power in kJ per sec}}{\text{Heat supplied in steam in kJ per sec}} = \frac{4.767}{40.36} = 0.1181 \text{ or 11.81%}
\]

Using eqn. (9.17), Brake thermal efficiency,

\[
\eta_b = \frac{\text{Heat equivalent of brake power in kJ per sec}}{\text{Heat supplied in steam in kJ per sec}} = \frac{3.607}{40.36} = 0.0894 \text{ or 8.94%}
\]

Problem-14: A single-cylinder, double-acting steam engine of 28 cm bore and 45 cm stroke works between a supply pressure of 980 kPa (9.8 bar) and a back pressure of 15 kPa (0.15 bar). Assuming a diagram factor of 0.7 and neglecting clearance volume, estimate the indicated power at 3 r.p.s. if the cut-off is at 1/3rd stroke.

If the above engine consumes 900 kg of dry saturated steam per hour, determine the indicated thermal efficiency of the engine.

Here, Expansion ratio, \( r = \frac{1}{\text{cut-off}} = \frac{1}{1/3} = 3 \), \( p_1 = 980 \text{ kPa} \), \( p_b = 15 \text{ kPa} \), and \( f = 0.7 \).

Actual m.e.p. \( p_m = \frac{p_1}{r} (1 + \log_e r) - p_b \)

\[
= 0.7 \left[ \frac{980}{3} (1 + \log_e 3) - 15 \right] = 469.37 \text{ kPa}
\]

Indicated power = \( 2 \times p_m \times a \times l \times N \)

\[
= 2 \times 469.37 \times 10^3 \times \frac{\pi}{4} \left( \frac{28}{100} \right)^2 \times \frac{45}{100} \times 3
\]

\[
= 77,980 \text{ W or 77.98 kW}
\]

Indicated thermal efficiency, \( \eta_i = \frac{\text{Heat equivalent of indicated power in kJ per sec}}{\text{Heat supplied in steam in kJ per sec}} \)

Heat equivalent of indicated power in kJ per sec = 77.98 kJ/sec.
Heat supplied in steam per sec. = \( m_s \) \( (H_1 - h_2) \) kJ/sec.

where, \( m_s \) = steam supplied per sec. = 900/3,600 = 0.25 kg.

\( H_1 \) = enthalpy of 1 kg of dry saturated steam at 9.8 bar = 2,777.3 kJ/kg, and

\( h_2 \) = enthalpy of 1 kg of water of exhaust steam at 0.15 bar = 225.94 kJ/kg.

Heat supplied in the steam/sec. = 0.25 \times (2,777.3 - 225.94) = 637.84 kJ/sec.

\[ \therefore \text{Indicated thermal eff., } \eta_i = \frac{77.98}{637.84} = 0.1222 \text{ or } 12.22\% \]

**Problem-15:** The following data were obtained during the trial of a single-cylinder, double-acting steam engine:

Engine speed, 4 r.p.s.; Cylinder diameter, 20 cm; Piston stroke, 30 cm; Indicated mean effective pressure (both ends), 1.2 bar; Effective brake radius, 60 cm; Net load on the brake wheel, 490 newtons; Pressure of steam supplied, 7 bar; Steam used per hour, 120 kg; Condensate temperature, 50°C. Steam supplied is dry saturated. Calculate:

(a) the mechanical efficiency, (b) the indicated thermal efficiency, (c) the brake thermal efficiency, and (d) the steam consumption in kg per kW-hr on brake power basis.

(a) Indicated power = \( 2 \times p_m \times a \times l \times N \)

\[ = 2 \times 120 \times 10^3 \times \frac{\pi}{4} \times (0.2)^2 \times 0.3 \times 4 = 9,042 \text{ W or } 9.042 \text{ kW} \]

Brake power = \( \frac{(W-S)R \times 2\pi N}{1,000} \)

\[ = \frac{490 \times 0.6 \times 2\pi \times 4}{1,000} = 7.392 \text{ kW} \]

Mechanical efficiency = \( \frac{\text{Brake power}}{\text{Indicated power}} = \frac{7.392}{9.042} = 0.8165 \text{ or } 81.65\% \)

(b) Indicated thermal efficiency, \( \eta_i = \frac{\text{Heat equivalent of indicated power in kJ per sec.}}{\text{Heat supplied in steam in kJ per sec.}} \)

Heat equivalent of indicated power = 9.042 kJ/sec.

Heat supplied in steam = \( m_s \) \( (H_1 - h_2) \) kJ/sec.

where \( H_1 \) = Enthalpy of 1 kg of steam at 7 bar from steam tables = 2,763.5 kJ/kg.

\( h_2 \) = Enthalpy of condensate/kg = \((50-0) \times 4.187 = 209.35 \text{ kJ/kg, and} \)

\( m_s \) = Steam supplied in kg per sec. = \( \frac{120}{3,600} \)

\[ \therefore \text{Heat supplied in steam,} = \frac{120}{3,600} (2,763.5 - 209.35) = 85.13 \text{ kJ/sec.} \]

\[ \therefore \text{Indicated thermal efficiency,} \eta_i = \frac{9.042}{85.13} = 0.1062 \text{ or } 10.62\% \]

(c) Brake thermal efficiency, \( \eta_b = \frac{\text{Heat equivalent of brake power in kJ per sec.}}{\text{Heat in steam supplied in kJ per sec.}} \)

\[ = \frac{7.392}{85.13} = 0.0868 \text{ or } 8.68\% \]

(d) Steam consumption on brake power basis in kg/kW-hr.
Problem-16: A single-cylinder, double-acting steam engine admits steam at a pressure of 800 kPa (8 bar) and temperature 200°C. The cut-off takes place at 0.4 of the stroke and engine runs at 200 r.p.m. Assuming a diagram factor of 0.85, calculate the indicated power of the engine when the cylinder diameter is 35 cm and stroke is 53 cm. The back pressure is 100 kPa (1 bar).

If the above engine consumes 1,960 kg of steam per hour, determine the indicated thermal efficiency of the engine. Take \( k_p \) for superheated steam as 2.1 kJ/kg K.

Here, Expansion ratio, \( r = 1/0.4 = 2.5 \), \( p_1 = 800 \) kPa, \( p_b = 100 \) kPa.

Hypothetical m.e.p. = \( \frac{p_1}{r} [1 + \log_e (r)] - p_b \)

= \( \frac{800}{2.5} [1 + \log_e (2.5)] - 100 = 513 \) kPa

Actual m.e.p., \( p_m = \) hypothethical m.e.p. \( \times f = 513 \times 0.85 = 436 \) kPa

Indicated power = \( 2 \times p_m \times a \times l \times N \)

= \( 2 \times 436 \times 10^3 \times \frac{\pi}{4} (0.35)^2 \times \frac{53}{100} \times \frac{200}{60} \)

= 1,482,200 W or = 148.2 kW

Heat equivalent of indicated power = 148.2 kJ/sec.

Mass of steam supplied in kg per second, \( m_s = \frac{1960}{3600} = 0.544 \) kg

From steam tables, at 8 bar, \( H_s = 2,769.1 \) kJ/kg, \( t_s = 170.43^\circ C \),

and at 1 bar, enthalpy of saturated water, \( h_2 = 417.46 \) kJ/kg.

\( H_1 = H_s + k_p (t_{sup} - t_s) = 2,769.1 + 2.1 (200 - 170.43) = 2,831.2 \) kJ/kg.

Heat supplied in steam in kJ per second = \( m_s \times (H_1 - h_2) \)

= 0.544 (2,831.2 - 417.46) = 1,313.1 kJ/sec.

Indicated thermal efficiency,

\[ \eta_i = \frac{\text{Heat equivalent of indicated power in kJ per sec.}}{\text{Heat supplied in steam in kJ per sec.}} = \frac{148.2}{1,313.1} = 0.113 \text{ or 11.3%} \]

Problem-17: Dry saturated steam at 8 bar (800 kPa) is admitted into the cylinder of a single-cylinder, double-acting steam engine. The cylinder diameter is 30 cm and stroke is 60 cm and cut-off is at 50% of the stroke. The back pressure is 1 bar (100 kPa). Taking the diameter of piston rod as 4 cm and assuming a diagram factor of 0.75 and mechanical efficiency of 70%, calculate the brake power of the engine at 250 r.p.m.

If the indicated thermal efficiency of the above engine is 15 per cent, calculate the steam consumption in kg per kW per hour on indicated power basis.

Here, Expansion ratio, \( r = 1/0.5 = 2 \), \( p_1 = 800 \) kPa, \( p_b = 100 \) kPa.

Hypothetical m.e.p. = \( \frac{p_1}{r} [1 + \log_e (r)] - p_b \)
\[
= \frac{800}{2} \left[1 + \log_e 2\right] - 100 = 578 \text{ kPa}
\]

\[\therefore \text{Actual m.e.p., } p_m = \text{theoretical m.e.p. } \times f = 578 \times 0.75 = 433 \text{ kPa (for each end)}\]

Area of cylinder (cover end) = \(\frac{\pi}{4} d^2 = \frac{\pi}{4}(30)^2 = 707 \text{ cm}^2\)

Effective area of cylinder (crank end) = \(\frac{\pi}{4}(d^2 - a^2) = \frac{\pi}{4}(30^2 - 4^2) = 695 \text{ cm}^2\)

Indicated power of engine = indicated power cover end + indicated power crank end

\[
= \left[433 \times 10^3 \times \frac{707}{10^4} \times \frac{60}{100} \times \frac{250}{60}\right] + \left[433 \times 10^3 \times \frac{695}{10^4} \times \frac{60}{100} \times \frac{250}{60}\right]
\]

\[= 76,500 + 75,200 = 1,51,700 \text{ W or 151.7 kW}\]

Brake power = Mechanical efficiency \times Indicated power = 0.7 \times 151.7 = 106.19 kW

From steam tables, at 8 bar, \(H_f = 2,769.1 \text{ kJ/kg, and}\)

\[\text{at 1 bar, heat remaining in condensate, } h_2 = 417.46 \text{ kJ/kg.}\]

Heat supplied in steam per second = \(m_s(H_f - h_2) = m_s (2,769.1 - 417.46) \text{ kJ/sec.}\)

Indicated thermal efficiency, \(\eta_i = \frac{\text{Heat equivalent of indicated power in kJ/sec.}}{\text{Heat supplied in steam in kJ/sec.}}\)

i.e. \(0.15 = \frac{151.7}{m_s(2,769.1 - 417.46)}\)

\[\therefore m_s = 0.43 \text{ kg/sec.}\]

\[\therefore \text{Steam consumption per hour} = 0.43 \times 3,600 = 1,548 \text{ kg/hour.}\]

Steam consumption in kg per kW per hour on indicated power basis,

\[= \frac{1,548}{151.7} = 10.2 \text{ kg/kW-hr.}\]

Problem-18: The following are the average readings taken during a trial on a steam engine plant:

Indicated Power, 30 kW; Brake Power, 22.5 kW; Pressure of steam supplied, 5.5 bar; Quality of steam supplied, 5% wet; Condenser vacuum, 6475 mm of Hg; Barometer reading, 760 mm of Hg; Steam consumption, 360 kg per hour; 45 kg of coal with a calorific value of 33,500 kJ/kg is supplied per hour. Calculate: (a) the mechanical and indicated thermal efficiencies of the steam engine, and (b) the overall efficiency of steam engine plant (from coal to brake).

(a) Mechanical efficiency, \(\eta_m = \frac{\text{Brake Power}}{\text{Indicated Power}} = \frac{22.5}{30} = 0.75\) or 75%

Mass of steam supplied per second, \(m_s = \frac{360}{3,600} = 0.1 \text{ kg}\)

At 5.5 bar, \(h = 655.93 \text{ kJ/kg and } L = 2,097 \text{ kJ/kg (from steam tables).}\)

Enthalpy of 1 kg of wet steam at 5.5 bar,

\[H_f = h_f + x_f L_f = 655.93 + 0.95 \times 2,097 = 2,648 \text{ kJ/kg.}\]
Condenser or back pressure = (760 - 647.5) = 133 kPa (\( \simeq \) 1 mm of Hg = 1.33 kPa)

At 15 kPa (0.15 bar), heat remaining in condensate,
\[ h_2 = 225.94 \text{ kJ/kg} \] (from steam tables).

Heat supplied in steam/sec. = \( m_s(H_1 - h_2) = 0.1 \times (2648 - 225.94) = 242.2 \text{ kJ/sec.} \)

Indicated thermal efficiency,
\[ \eta_i = \frac{\text{Heat equivalent of Indicated power in kJ/sec.}}{\text{Heat supplied by steam in kJ/sec.}} = \frac{30}{242.2} = 0.1239 \text{ or } 12.39\% \]

(b) Using eqn. (9.19), overall efficiency of the steam engine plant
\[ = \frac{\text{Heat equivalent of brake power in kJ/sec.}}{\text{Heat supplied by coal in kJ/sec.}} \]
\[ = \frac{\text{Brake power in kW}}{m_f \times C.V.} \]

where \( m_f \) = Mass of coal supplied in kg per sec., and
\( C.V. \) = Calorific value of coal used in the boiler in kJ/kg.

\[ \therefore \text{Overall efficiency of steam engine plant} = \frac{22.5}{45 \times 33,500} = 0.0537 \text{ or } 5.37\% \]

9.10 Heat Engine Cycle

A heat engine cycle is a series of thermodynamic processes through which a working fluid or substance passes in a certain sequence. At the completion of the cycle, the working fluid returns to its original thermodynamic state, i.e. the working fluid at the end of cycle has the same pressure, volume, temperature and internal energy that it had at the beginning of the cycle. Somewhere during every cycle, heat is received by working fluid. It is then the object of the heat engine cycle to convert as much of this heat energy as possible into useful work. The heat energy which is, thus, not converted, is rejected by the working fluid during some process of the cycle.

Any machine designed to carry out a thermodynamic cycle and thus to convert heat energy supplied to it into mechanical energy, is called a heat engine. Hence, the cycle it operates on is known as a heat engine cycle.

The amount of heat which is transformed into mechanical energy is known as available energy of the cycle. It is equal to the difference between the heat supplied and the heat rejected in the absence of any other losses. This statement is of course a direct consequence of the law of conservation of energy.

Let \( Q \) = available energy for doing work per cycle,
\[ Q_1 = \text{heat received during each cycle, and} \]
\[ Q_2 = \text{heat rejected during each cycle.} \]

Then, \( Q = Q_1 - Q_2 \)

If \( W = \text{net work done during the cycle,} \)
Then, \( W = Q = Q_1 - Q_2 \)

The efficiency of a heat engine cycle is defined as the ratio of the available heat energy of the cycle for during work, and the heat received during the cycle. Thus,
Efficiency, $\eta = \frac{\text{Heat equivalent of the net work of the cycle}}{\text{Heat received during the cycle}}$

\[ \eta = \frac{Q}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1} \]  \hspace{1cm} (9.20)

The definition of efficiency given above is applicable to any type of heat engine cycle. Hence, the expression for efficiency given by the eqn. (9.20) is known as theoretical thermal efficiency of the cycle, as it does not take into account any practical losses which do occur in the actual running of the engine.

9.11 Steam Power Cycles

Heat engine cycles, using steam as the working fluid, are Carnot and Rankine cycles. Carnot cycle is a theoretical cycle which serves as a yard-stick for comparison purposes. All actual steam engines operate on theoretical cycle, known as the Rankine cycle.

9.11.1 Carnot Cycle: The cycle is represented in fig. 9-11 on $T-\phi$ and $p-v$ diagrams. It consists of following four operations — two constant pressure (isothermal) operations $(a-b)$ and $(c-d)$, and two frictionless adiabatic or isentropic operations $(b-c)$ and $(d-a)$. 

— Heat is supplied at constant temperature $T_1$ and constant pressure $p_1$, where water at its saturation temperature $T_1$ is heated to form wet steam of dryness fraction $x_1$. This operation (isothermal expansion) is represented by $(a-b)$.

— In the next operation $(b-c)$, this steam is expanded isentropically to temperature $T_2$ and pressure $p_2$. The condition of steam after expansion is represented by point $c$.

— Then heat is rejected at constant pressure $p_2$ and constant temperature $T_2$. Steam becomes wetter as it is exhausted and cooled from $c$ to $d$. This operation (isothermal compression) is represented by $(c-d)$.

— Lastly, the wet steam at $d$ is compressed isentropically till the steam comes to its initial state of temperature $T_1$ and pressure $p_1$. This operation is represented by $(d-a)$.

Thus, the cycle is completed.
Referring to the $T - \Phi$ diagram of fig. 9-11(a), heat supplied at constant temperature $T_1$ during $(a-b)$ is represented by area $a-b-f-e$ and is equal to $T_1 (\Phi_b - \Phi_a)$ or $T_1 (\Phi_c - \Phi_d)$.

The amount of heat rejected $(c-d)$ at constant temperature $T_2$ is represented by area $c-d-e-f$ and is equal to $T_2 (\Phi_c - \Phi_d)$.

As there is no exchange of heat during isentropic operations $(b-c)$ and $(d-a)$, Net work done = heat supplied during operation $(a-b)$ - heat rejected during operation $(c-d)$

$= T_1 (\Phi_c - \Phi_d) - T_2 (\Phi_c - \Phi_d) = (T_1 - T_2) (\Phi_c - \Phi_d)$.

\[ \therefore \text{Efficiency of the Carnot cycle} = \frac{\text{work done}}{\text{heat supplied}} = \frac{(T_1 - T_2) (\Phi_c - \Phi_d)}{T_1 (\Phi_c - \Phi_d)} = \frac{T_1 - T_2}{T_1} \qquad (9.21) \]

Although Carnot cycle is thermodynamically simple, yet it is extremely difficult to operate in practice, because the isothermal compression $(c-d)$ must be stopped at $d$, so that subsequent isentropic compression $(d-a)$ restores the fluid to its initial state $a$.

If superheated steam is used, the cycle would be still more difficult to operate in practice, owing to the necessity of supplying the superheat at constant temperature instead of constant pressure, as is customary. In a practical cycle, limits of pressure and volume are far more easily realised than limits of temperature so that no practical engine operates on the Carnot cycle, although all modern cycles aspire to achieve it as it is more efficient than Rankine cycle. So it is an ideal cycle for comparison purposes.

9.11.2 Rankine Cycle: In a steam plant, the supply of heat and rejection of heat is more easily performed at constant pressure than at constant temperature, and although engines have operated on this principle since the time of Watt, yet it was not until 1844, that an attempt was made by Rankine to calculate the maximum possible work that could be developed by an engine using dry saturated steam, between the pressure limits of the boiler and the condenser. Two years later, Clausius developed a more general expression for the maximum thermal efficiency of a steam engine by allowing for the steam being wet initially.

The ideal Rankine cycle is used as a yard-stick for the determination of the best performance obtainable in a simple steam engine cycle operating under the specific steam pressure.

Rankine cycle is a modified Carnot cycle. Except for the isentropic compression $(d-a)$ on the Carnot cycle (fig. 9-11), the Rankine and Carnot cycles, are the same.

The ideal Rankine cycle is represented on $T - \Phi$ diagram in fig. 9-13. A flow diagram of a condensing steam plant working on the Rankine cycle is shown in fig. 9-12.

The following assumptions are made in the working of the Rankine cycle:
— The working fluid (water) is pumped into the boiler, evaporated into steam in the boiler, expanded in the prime-mover (steam engine or steam turbine), condensed in the condenser and returned to the feed pump and circulated again in a closed circuit.

— Heat is added only in the boiler and is rejected only in the condenser. There is no transfer of heat between the working fluid and the surroundings at any place except the boiler and the condenser.

— There is no pressure drop in the piping system.

— Expansion in the prime-mover occurs without friction or heat transfer, i.e. the expansion is isentropic, in which case the entropy of the working fluid entering and leaving the prime-mover is same.

— The working fluid (water) is not undercooled in the condenser, i.e. temperature of the water leaving the condenser is same as the saturation temperature corresponding to the exhaust pressure.

In the Rankine cycle, we commence with one kilogram of water at the lower temperature and pressure $T_2$ and $p_2$ respectively as shown at point $a$ in fig. 9-13. The pressure of the water is then raised to $p_1$ by frictionless, adiabatic compression $(a-b)$ in the feed pump. The increase in temperature consequent on this compression may be of the order of a few degrees, and is represented by vertical line $(a-b)$ on $T - \Phi$ diagram. From $b$ to $c$ the liquid receives sensible heat at constant pressure $p_1$ (temperature increases to $T_1$) to be followed by evaporation at constant pressure $p_1$, which may be partial at $d$ (i.e. wet steam) or complete at $d'$ (i.e. dry saturated steam) or frequently a superheat is imparted to raise the temperature at constant pressure $p_1$ to $T_{sup}$ (which is located by point $d''$).

From $d$, $d'$ or $d''$ the steam expands isentropically to $e$, $e'$ or $e''$ respectively until its pressure becomes $p_2$ and temperature $T_2$. The last operation $(ea$, $e'a$ or $e''a)$ is condensation at constant temperature $T_2$ and constant pressure $p_2$ (i.e. isothermal compression), until the working fluid is returned to its original state at $a$.

The process $abc$ has been greatly exaggerated on the $T - \Phi$ diagram (fig. 9-13) to illustrate the process. If plotted actually, it is difficult to differentiate between $abc$ and $ac$ (along the water line), and in normal practice, it is assumed to be $ac$.

During the operation of the cycle, heat is added in the boiler, the work is performed in the prime-mover (steam engine), heat is rejected in the condenser, and the work is done on water as it passes through the feed pump. The heat equivalent of work done in pumping feed-water into the boiler is so small that it is usually neglected, i.e. triangular strip $abc$ in the liquid region is neglected.
Let \( H_1 \) = Enthalpy of steam at pressure \( p_1 \) as it enters the prime-mover at condition \( d, d' \) or \( d'' \).

\( H_2 \) = Enthalpy of steam at pressure \( p_2 \) as it leaves the prime-mover at condition \( e, e' \) or \( e'' \), and

\( h_2 \) = Enthalpy of water at pressure \( p_2 \) as it enters the feed pump at condition \( a \).

Work done by the prime-mover supplied with wet steam (at condition \( d \))

\[ = H_d - H_e = H_1 - H_2 \text{ kJ per kg of steam.} \]

Heat rejected in the condenser = \( H_e - H_a = H_2 - h_2 \text{ kJ per kg of steam.} \)

Heat supplied = work done + heat rejected

\[ = (H_1 - H_2) + (H_2 - h_2) = H_1 - h_2 \text{ kJ per kg of steam.} \]

Efficiency of the Rankine cycle = \( \frac{\text{Work done in kJ per kg of steam}}{\text{Heat supplied in kJ per kg of steam}} \)

\[ = \frac{H_1 - H_2}{H_1 - h_2} \quad \ldots (9.22) \]

From the given values of \( p_1, p_2 \) and condition of steam before entering the prime-mover (at point \( d \)), the condition of steam after isentropic expansion \( d-e \) at point \( e \) (fig. 9-13) is evaluated by equating entropies at \( d \) and \( e \) and using entropy values from steam tables or can be directly obtained from \( H - \Phi \) chart by drawing vertical line as shown in fig. 9-21.

The Rankine cycle, ignoring the effect of feed pump work, is represented by \( p - v \) diagram in fig. 9-14. The work done per cycle is given by area \( a-b-c-d \). This area can be divided in three parts viz. work done by steam during admission \((b-c)\) work done by steam during isentropic expansion \((c-d)\), and work done on the steam during exhaust \((d-a)\). Thus,

Work done by steam on the piston during admission \((b-c) = p_1 v_1 \)

Work done during isentropic expansion \((c-d) = \frac{p_1 v_1 - p_2 v_2}{n-1} \)

Work done on the steam during exhaust stroke \((d-a) = p_2 v_2 \)

\[ \text{Net work done during the cycle} \]

\[ = p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n-1} - p_2 v_2 \]

\[ = (p_1 v_1 - p_2 v_2) \left(1 + \frac{1}{n-1}\right) \]

\[ = \frac{n}{n-1} (p_1 v_1 - p_2 v_2) \]\n
kJ per kg of steam \( \ldots (9.23) \)

where \( p_1 \) and \( p_2 \) are in kPa and \( v_1 \) and \( v_2 \) are in \( m^3/kg \).

The value of \( n \) can be evaluated from equation given by Dr. Zeuner or may be taken as 1.135 for steam initially
wet and 1.3 for steam initially superheated.

Alternatively, the work done during isentropic expansion (c-d) can be found from the definition of non-flow adiabatic process viz.

Work done during expansion \( = \) change of internal energy during expansion \( = u_1 - u_2 \)
\[ = (H_1 - p_1v_1) - (H_2 - p_2v_2) = H_1 - H_2 - (p_1v_1 - p_2v_2) \text{ kJ per kg of steam} \]

\[ \therefore \text{Net work done during the cycle} = p_1v_1 + [H_1 - H_2 - (p_1v_1 - p_2v_2)] - p_2v_2 \]
\[ = H_1 - H_2 \text{ kJ/kg (same as before).} \]

Heat equivalent of one kW-hour \( = 3,600 \text{ kJ (since 1 kW = 1 kJ/sec.)} \)

Work done per kg of steam in Rankine engine \( = H_1 - H_2 \text{ kJ} \)

Steam consumption \( = \frac{\text{Heat equivalent of one kW-hour in kJ}}{\text{Work done per kg of steam in kJ}} \)
\[ = \frac{3,600}{H_1 - H_2} \text{ kg per kW-hour} \quad \ldots \text{(9.24)} \]

9.11.3 Modified Rankine Cycle : In practice, it is not economical to expand the steam to the extreme toe of the \( p-v \) diagram, which is represented by the point \( d \) (fig. 9-15). It is obvious that the diagram is very narrow at the toe. For this reason, the expansion is terminated (stopped) at point \( e \) and the cylinder is connected either to the atmosphere or to the condenser through exhaust port. Steam rushes out of the cylinder, causing drop in pressure up to the exhaust pressure. The expansion of the steam is thus completed by the constant volume line \( e-f \) as shown in fig. 9-15(a).

The loss of work due to incomplete expansion is represented by the area \( e-f-d \). This cutting-off the toe occurs in reciprocating steam engine cylinder where the piston stroke is limited. In fact, the extra work (area \( e-f-d \)) obtained by complete expansion is not sufficient to overcome the friction of the moving parts of the engine.

![Pressure-Volume Diagram](a) \( p-v \) diagram

![Temperature-Entropy Diagram](b) \( T-\phi \) diagram

Fig. 9-15. Modified Rankine cycle.

The constant volume expansion \( e-f \) after the end of isentropic expansion \( c-e \) can be represented on the \( T-\phi \) diagram as shown in fig. 9-15(b) by considering it as partial condensation. There will, therefore, be a reduction in dryness fraction during the operation \( e-f \).
As the value of specific volume of steam at point $e$ is known, the value of dryness fraction for any other pressure between $e$ and $f$ may be calculated.

The work done during the modified Rankine cycle shown in fig. 9-15(a), may be calculated from the area $a-b-c-e-f$ of the $p-v$ diagram.

Let $p_1$, $v_1$, $H_1$ and $u_1$ apply to initial condition of steam at $c$.
$p_2$, $v_2$, $H_2$ and $u_2$ apply to condition of steam at $e$, and
$p_3$, $h_3$ apply to condition of water at $a$.

Then, work done during cycle per kg of steam

$$= \text{area } a-b-c-e-f \text{ of fig. 9-15(a)}$$

$$= \text{area } b-c-g-o \text{ plus area } c-e-h-g \text{ minus area } a-f-h-o$$

$$= \text{Work done during admission (b-c) plus work done during isentropic expansion (c-e) minus work done during exhaust (f-a)}$$

$$= p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n-1} - p_3 v_2$$

Alternatively, writing the work done during isentropic expansion $c-e$ as $u_1 - u_2$, i.e. change of internal energy,

Work done during cycle per kg of steam

$$= p_1 v_1 + (u_1 - u_2) - p_3 v_2 \text{ kJ}$$

where $p_1$ and $p_3$ are in kPa, and $v_1$ and $v_2$ are in m$^3$/kg

Heat supplied per kg of steam = $H_1 - h_3$ kJ

Modified Rankine cycle efficiency,

$$= \frac{\text{Work done during cycle in kJ per kg of steam}}{\text{Heat supplied in kJ per kg of steam}}$$

$$= \frac{p_1 v_1 + (u_1 - u_2) - p_3 v_2}{H_1 - h_3} \quad \ldots (9.25)$$

Fig. 9-16. $p-v$ diagram of the modified Rankine cycle. Alternatively, referring to fig. 9-16, Work done during cycle per kg of steam

$$= \text{area } a-b-c-d-e-f \text{ plus area } b-c-e-k \text{ plus area } k-e-f-a$$

$$= (H_c - H_a) + [(p_e - p_l) v_e] \text{ kJ/kg} \quad \ldots (9.26a)$$

where $p_e = \text{Pressure at release in kPa}$, $p_l = \text{Back pressure in kPa}$,
$H_c = \text{Enthalpy of steam at inlet (before isentropic expansion) in kJ/kg}$,
$H_a = \text{Enthalpy of steam after isentropic expansion in kJ/kg}$, and
$v_e = \text{Volume occupied by steam at the end of isentropic expansion i.e. at } e \text{ in m}^3/\text{kg}$.

Heat supplied per kg of steam = $H_c - h_a$ kJ/kg

where $h_a = \text{enthalpy (sensible heat) of 1 kg of water of exhaust steam (condensate)}$ in kJ/kg.
Efficiency of modified Rankine cycle = \( \frac{\text{Work done per kg of steam in kJ}}{\text{Heat supplied per kg of steam in kJ}} \)

\[ = \frac{(H_c - H_a) + (p_e - p_l) v_e}{H_e - h_a} \] \hspace{1cm} (9.26b)

9.11.4 Relative Efficiency: The relative efficiency of a steam engine is the ratio between the indicated thermal efficiency and Rankine cycle efficiency operating between the same pressure limits. It is also known as efficiency ratio.

Rankine cycle efficiency = \( \frac{H_1 - H_2}{H_1 - h_2} \) [from eqn. (9.22)]

Thus, Relative efficiency = \( \frac{\text{Indicated thermal efficiency}}{\text{Rankine cycle efficiency}} \) \hspace{1cm} (9.27)

When brake thermal efficiency is used instead of indicated thermal efficiency in eqn. (9.27), the efficiency is termed overall efficiency ratio.

The efficiency ratio for a given steam engine indicates the percentage of the available heat converted into mechanical work. It also provides a measure of the excellence of design of the steam engine for the steam conditions under which it is operating and of its mechanical condition.

Problem-19: A Carnot engine receives dry saturated steam at 35 bar (3,500 kPa) and exhausts at 0.7 bar (70 kPa). Calculate: (a) the heat supplied per kg of steam, (b) the heat rejected per kg of steam, (c) the maximum work obtained per kg of steam, and (d) the Carnot cycle efficiency.

From steam tables:

<table>
<thead>
<tr>
<th>( p ) bar</th>
<th>( t_s ) °C</th>
<th>( \Phi_w ) kJ/kg K</th>
<th>( \Phi_e ) kJ/kg K</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>242.6</td>
<td>2.7253</td>
<td>6.1253</td>
</tr>
<tr>
<td>0.7</td>
<td>89.95</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

\[ T_1 = 242.6 + 273 = 515.6 \text{ K}; \quad T_2 = 89.95 + 273 = 362.95 \text{ K} \]

(a) T-\( \phi \) diagram \hspace{1cm} (b) P-\( \eta \) diagram
Referring to fig. 9-17(a),

(a) Heat supplied = \( T_1 (\Phi_b - \Phi_d) = 515.6 \times (6.1253 - 2.7253) = 1,753.04 \text{ kJ/kg} \)

(b) Heat rejected = \( T_2 (\Phi_c - \Phi_d) \]
\[
\begin{align*}
&= 362.95 \times (6.1253 - 2.7253) \\
&= 362.95 \times 3.4 = 1,234.03 \text{ kJ/kg}
\end{align*}
\]

(c) Work obtained = Heat supplied – Heat rejected
\[
1,753.04 - 1,234.03 = 519.01 \text{ kJ/kg}
\]

(d) Carnot cycle efficiency or thermal efficiency
\[
\frac{\text{Work done per kg}}{\text{Heat supplied per kg}} = \frac{519.01}{1,753.04} = 0.2961 \text{ or } 29.61\% 
\]

Alternatively, using eqn. (9.21), Carnot cycle efficiency
\[
\frac{T_1 - T_2}{T_1} = \frac{515.6 - 362.95}{515.6} = 0.2961 \text{ or } 29.61\% \text{ (same as before)}
\]

**Problem-20**

A steam power plant is supplied with dry saturated steam at a pressure of 14 bar and exhausts into a condenser at a pressure of 0.3 bar. Using the steam tables, calculate the efficiency of Carnot cycle and Rankine cycle for these conditions. Draw \( T - \Phi \) diagrams for the cycles.

From steam tables:

<table>
<thead>
<tr>
<th>( p ) bar</th>
<th>( t_e ) °C</th>
<th>( h ) kJ/kg</th>
<th>( L ) kJ/kg</th>
<th>( H ) kJ/kg</th>
<th>( \Phi_w ) kJ/kg K</th>
<th>( \Phi_a ) kJ/kg K</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>195.07</td>
<td>—</td>
<td>—</td>
<td>2,790</td>
<td>—</td>
<td>6493</td>
</tr>
<tr>
<td>0.3</td>
<td>69.10</td>
<td>289.23</td>
<td>2,336.1</td>
<td>—</td>
<td>0.9439</td>
<td>7.7686</td>
</tr>
</tbody>
</table>

**Carnot cycle**: \( T_1 = 195.07 + 273 = 468.07 \) K, \( T_2 = 69.1 + 273 = 342.1 \) K

Referring to fig. 9-18 (a) and using eqn. (9.21),

Carnot cycle efficiency = \( \frac{T_1 - T_2}{T_1} = \frac{468.07 - 342.1}{468.07} = 0.2691 \text{ or } 26.91\% \)

\[\text{(a) Carnot cycle} \quad \text{Fig. 9-18.} \quad \text{(b) Rankine cycle}\]
Rankine cycle: Referring to fig. 9-18 (b) and considering isentropic expansion c-d, Entropy at c = Entropy at d i.e. \( \Phi_c = \Phi_d + x_d(\Phi_{sd} - \Phi_{wd}) \)
i.e. \( 6.4693 = 0.9439 + x_d(7.7686 - 0.9439) \) \( \therefore x_d = 0.81 \)
\( H_d = h_d + x_dL_d = 289.23 + 0.81 \times 2,336.1 = 2,181.47 \text{ kJ/kg} \)
\( H_c = 2,790 \text{ kJ/kg (from steam tables)} \)
Work done = \( H_c - H_d = 2,790 - 2,181.47 = 608.53 \text{ kJ/kg} \)
Heat supplied = \( H_c - h_b = 2,790 - 289.23 = 2,500.77 \text{ kJ/kg} \)
Using eqn. (9.22), Rankine cycle efficiency
\[
\text{efficiency} = \frac{\text{work done in kJ per kg}}{\text{heat supplied in kJ per kg}} = \frac{608.53}{2,500.77} = 0.2433 \text{ or } 24.33% 
\]

Alternatively, the value of \( x_d \) and \( H_c - H_d \) can be directly obtained from \( H - \Phi \) chart by drawing vertical line from the point on dry saturated line intersected by 14 bar line, to meet the line of 0.3 bar. The method of finding out \( x_d \) and \( H_c - H_d \) from \( H - \Phi \) chart is shown in fig. 9-21. Hence, the Rankine cycle efficiency.

It may be noted that the Carnot cycle is more efficient than Rankine cycle.

Problem-21: In a Carnot cycle the upper and lower limits of temperature correspond to steam pressures of 28 bar and 0.15 bar respectively. Dry saturated steam is supplied to the engine. Evaluate: (a) the dryness fraction of steam at the beginning of isentropic compression, (b) the work done per kg of steam, (c) the heat supplied per kg of steam, and (d) the Carnot cycle efficiency. Compare items (b), (c) and (d) with the corresponding figures obtained for an engine working on the Rankine cycle between the same limits of pressure and temperature.

From steam tables:

<table>
<thead>
<tr>
<th>( \rho \text{ bar} )</th>
<th>( t ^\circ C )</th>
<th>( h \text{ kJ/kg} )</th>
<th>( L \text{ kJ/kg} )</th>
<th>( H \text{ kJ/kg} )</th>
<th>( \Phi_w \text{ kJ/kg K} )</th>
<th>( \Phi_s \text{ kJ/kg K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>230.1</td>
<td>990.59</td>
<td>—</td>
<td>2,804</td>
<td>2,6109</td>
<td>6,2139</td>
</tr>
<tr>
<td>0.15</td>
<td>53.97</td>
<td>225.94</td>
<td>2,373.1</td>
<td>—</td>
<td>0.7549</td>
<td>8,0085</td>
</tr>
</tbody>
</table>

(a) Carnot cycle  
(b) Rankine cycle

Fig. 9-19. T-\( \Phi \) diagram.
Carnot cycle:

(a) $T_1 = 230.1 + 273 = 503.1$ K; $T_2 = 53.97 + 273 = 326.97$ K

Referring to $T-\Phi$ diagram of fig. 9-19(a) and considering isentropic compression $d-a$

Entropy at $a = $ Entropy at $d$ i.e. $\Phi_{wa} = \Phi_{wd} + x_d (\Phi_{sd} - \Phi_{wd})$

i.e. $2.6109 = 0.7549 + x_d (8.0085 - 0.7549) \therefore x_d = 0.256$

(b) Work done = $(T_1 - T_2) (\Phi_{sc} - \Phi_{wd})$

= $(T_1 - T_2) (\Phi_{sb} - \Phi_{wa})$

= $(503.1 - 326.97) (6.2139 - 2.6109) = 634.6$ kJ/kg.

(c) Heat supplied = $H_b - h_a = 2.804 - 990.59 = 1,813.41$ kJ/kg

(d) Carnot cycle efficiency = \[
\frac{\text{work done}}{\text{heat supplied}} = \frac{634.6}{1,813.41} \times 100 = 35\%
\]

Alternatively, using eqn. (9.21),

Carnot cycle efficiency = \[
\frac{T_1 - T_2}{T_1} = \frac{503.1 - 326.97}{503.1} \times 100 = 35\% \text{ (same as before)}
\]

Rankine cycle:

(b) Referring to fig. 9-19(b), Entropy at $b = $ Entropy at $c$

i.e. $\Phi_{sb} = \Phi_{wc} + x_c (\Phi_{sc} - \Phi_{wc})$

i.e. $6.2139 = 0.7549 + x_c (8.0085 - 0.7549)$

$\therefore x_c = 0.753$ (dryness fraction at $c$)

Now, $H_c = h_c + x_c L_c = 225.94 + 0.753 \times 2,373.1 = 2,012.88$ kJ/kg

Work done = $H_b - H_c = 2.804 - 2,012.88 = 791.12$ kJ/kg of steam.

(c) Heat supplied = $H_b - h_d = 2.804 - 225.94 = 2,578.06$ kJ/kg of steam.

(d) Rankine cycle efficiency = \[
\frac{\text{work done in kJ/kg}}{\text{heat supplied in kJ/kg}} = \frac{791.12}{2,578.06} \times 100 = 30.7\%
\]

Problem-22: Steam at 14 bar and 315°C is supplied to an engine working on the Rankine cycle. The steam is exhausted at 0.07 bar. Draw the $T-\Phi$ and $p-v$ diagrams for the cycle and calculate using the steam tables: (a) the condition of steam after isentropic expansion, (b) the Rankine cycle efficiency, (c) the mean effective pressure, (d) the ideal steam consumption per kW hour, and (e) the actual steam consumption per kW hour of the engine if the relative efficiency is 0.6. Take $k_p$ for superheated steam as 2.1 kJ/kg K.

From steam tables, at 14 bar, $H = 2,790$ kJ/kg, $\Phi_s = 6.4693$ kJ/kg K, .. 195.07°C, and at 0.07 bar, $h = 163.4$ kJ/kg, $L = 2,409.1$ kJ/kg, $v_s = 20.53$ m³/kg, $\Phi_w = 0.5592$ kJ/kg K and $\Phi_s = 8.2758$ kJ/kg K.

(a) Referring to $T-\Phi$ diagram of fig. 9-20 and considering isentropic expansion $c-d$,

Entropy at $c = $ Entropy at $d$

i.e. $\Phi_{sc} + k_p \log_e \left(\frac{T_{supc}}{T_{satc}}\right) = \Phi_{wd} + x_d (\Phi_{sd} - \Phi_{wd})$

i.e. $6.4693 + 2.1 \log_e \left(\frac{315 + 273}{195.07 + 273}\right) = 0.5592 + x_d (8.2758 - 0.5592) \therefore x_d = 0.828$
STEAM ENGINES

**: x_d = 0.828**

(a) T-\(\phi\) diagram

(b) \(P-v\) diagram

Fig. 9-20. Rankine cycle diagrams.

(b) \(H_c = 2,790 + 2.1 \times (315 - 195.07) = 3,041.85\) kJ/kg.

\(H_d = 163.4 + 0.828 \times 2,409.1 = 2,158\) kJ/kg

\(h_a = 163.4\) kJ/kg (from steam tables.).

Work done = \(H_c - H_d = 3,041.85 - 2,158 = 883.85\) kJ/kg.

Heat supplied = \(H_c - h_a = 3,041.85 - 163.4 = 2,878.45\) kJ/kg

Now, Rankine cycle efficiency:

\[
\text{work done in kJ per kg} = \frac{883.85}{2,878.45} \times 100 = 30.7\% 
\]

(c) Specific volume, \(v_s\) at 0.07 bar = 20.53 m\(^3\)/kg (from steam tables).

Volume, \(v_d = x_d \times v_s = 0.828 \times 20.53\) m\(^3\)/kg.

Mean effective pressure = \(\frac{\text{work done in kJ or kN.m per kg of steam}}{\text{stroke volume in m}^3\text{ per kg of steam (v_d)}}\)

\[
= \frac{883.85}{0.828 \times 20.53} = 52 \text{ kPa or } 52 \text{ kN/m}^2
\]

(d) Work done in Rankine cycle = \(H_c - H_d = 3,041.85 - 2,158 = 883.85\) kJ/kg.

Using eqn. (9.24) ideal steam consumption in kg per kW-hr.

\[
= \frac{\text{Heat equivalent of one kW-hr.}}{\text{Work done per kg of steam}} = \frac{1 \times 3,600}{H_c - H_d} = \frac{3,600}{883.85} = 4.078 \text{ kg per kW-hr.}
\]

(e) Using eqn. (9.27), Relative efficiency = \(\frac{\text{Indicated thermal efficiency}}{\text{Rankine cycle efficiency}}\)

\[
= \frac{\text{Indicated thermal efficiency}}{\text{Indicated thermal efficiency}} = \frac{0.6 \times 0.307}{0.1842} = 0.18-42\% 
\]

Indicated thermal efficiency, \(\eta_i = \frac{\text{Heat equivalent of one kW-hr in kJ}}{M_s (H_c - h_a) \text{ in kJ}}\)

(where \(M_s\) is the actual steam consumption in kg per kW-hr.)
i.e. \( 0.1842 = \frac{1 \times 3,600}{M_s \times (3,041.85 - 163.4)} \)
\[ \therefore M_s = \frac{1 \times 3,600}{0.1842 \times (3,041.85 - 163.4)} = 6.79 \text{ kg per kW-hr.} \]

Alternatively, \( M_s = \frac{4.078}{0.6} = 6.79 \text{ kg per kW-hr. (same as before)} \)

**Problem-23** : A steam engine is supplied with dry saturated steam at 7 bar and exhausts at 1.4 bar. The steam consumption was found to be 2 kg per minute when developing indicated power of 4.5 kW. Using the steam tables or \( H - \Phi \) chart, find the relative efficiency.

From steam tables, at 7 bar, \( H = 2,763.5 \text{ kJ/kg, } \Phi_s = 6.7080 \text{ kJ/kg K;} \) and at 1.4 bar, \( h = 458.39 \text{ kJ/kg, } L = 2,232.1 \text{ kJ/kg, } \Phi_w = 1.4109 \text{ kJ/kg K and } \Phi_s = 7.2464 \text{ kJ/kg K.} \)

**Indicated thermal efficiency,** \( \eta_i = \frac{\text{Heat equivalent of indicated power in kJ per sec.}}{\text{Net heat supplied in kJ per sec.}} \)
\[ = \frac{m_s (H_1 - h_2)}{2,763.5 - 458.39} = 0.0587 \text{ or } 5.87\% \]

In order to determine the relative efficiency or efficiency ratio, it is first necessary to determine the Rankine cycle efficiency operating between same pressure limits, i.e. 7 bar and 1.4 bar.

Using the steam tables and considering isentropic expansion 1 - 2, as shown in fig. 9-21.

Entropy before expansion, \( \Phi_1 \)
\[ = \text{Entropy after expansion, } \Phi_2 \]
\[ \Phi_s1 = \Phi_w2 + x_2 (\Phi_s2 - \Phi_w2) \]
\[ \text{i.e. } 6.7080 = 1.4109 + x_2 (7.2464 - 1.4109) \]
\[ \therefore x_2 = 0.908 \]

Thus, enthalpy after isentropic expansion,
\[ H_2 = h_2 + x_2L_2 \]
\[ = 458.39 + 0.908 \times 2,232.1 = 2,485.14 \text{ kJ/kg} \]

Enthalpy before isentropic expansion, \( H_1 = 2,763.5 \text{ kJ/kg.} \)
\[ \therefore \text{Isentropic enthalpy drop = Work done} \]
\[ = H_1 - H_2 = 2,763.5 - 2,485.14 = 278.36 \text{ kJ/kg of steam.} \]

Alternatively, the value of \( H_1 - H_2 \) (isentropic enthalpy drop) can be directly obtained from \( H - \Phi \) chart by drawing vertical line 1-2 from point on saturation line intersected by 7 bar line, to meet line of 1.4 bar as shown in fig. 9-21.

Heat supplied = \( H_1 - h_2 = 2,763.5 - 458.39 = 2,305.1 \text{ kJ/kg of steam.} \)

**Rankine cycle efficiency =** \( \frac{\text{Work done in kJ per kg of steam}}{\text{Heat supplied in kJ per kg of steam}} \)
Using eqn. (9.27), Relative efficiency or efficiency ratio

\[
\frac{\text{Indicated thermal efficiency}}{\text{Rankine cycle efficiency}} = \frac{0.0587}{0.12077} = 0.486 \text{ or } 48.6\%
\]

**Problem-24:** In a modified Rankine cycle steam is supplied at 5.5 bar and 190°C. It expands isentropically to 2.8 bar when it is released at constant volume to exhaust pressure of 1 bar.

Using the steam tables only, determine: (a) the work done per kg of steam supplied, (d) the modified Rankine cycle efficiency, (c) the ideal steam consumption per kW-hr and (d) the volume of steam to be supplied per kW-hr. Take kp of superheated steam as 2.1 kJ/kg K.

From steam tables, at 5.5 bar, \( \Phi_s = 6.7893 \text{ kJ/kg K} \), \( H = 2,753.0 \text{ kJ/kg} \), \( v_s = 0.3427 \text{ m}^3/\text{kg} \), \( t_s = 155.48°C \); at 2.8 bar, \( \Phi_w = 1.6472 \text{ kJ/kg K} \), \( \Phi_s = 7.0149 \text{ kJ/kg K} \), \( h = 551.48 \text{ kJ/kg} \), \( L = 2,170.7 \text{ kJ/kg} \), \( v_s = 0.6463 \text{ m}^3/\text{kg} \), and at 1 bar, \( h = 417.46 \text{ kJ/kg} \).

(a) Referring to fig. 9-22, and considering isentropic expansion c-e,

\[ H_c = H_a + \frac{2.1 \log_e \left( \frac{190 + 273}{155.48 + 273} \right)}{x_e} \]

\[ = 2,825.5 \text{ kJ/kg} \]

\[ H_e = h_e + x_e L_e \]

\[ = 551.48 + 0.988 \times 2,170.7 \]

\[ = 2,696 \text{ kJ/kg} \]

\[ V_e = x_e \times v_s \]

\[ = 0.988 \times 0.6463 = 0.6385 \text{ m}^3/\text{kg} \]

Using eqn. (9.26a), work done = \( (H_c - H_e) + [(p_e - p_i) v_e] \)

\[ = 2,825.5 - 2,696 + [(280 - 100) \times 0.6385] \]

\[ = 244.43 \text{ kJ/kg of steam} \]

Heat supplied = \( H_c - h_a = 2,825.5 - 417.46 = 2,408.04 \text{ kJ/kg of steam} \)

(b) Using eqn. (9.26b), Modified Rankine cycle efficiency

\[ \frac{\text{work done in kg per kg of steam}}{\text{heat supplied in kJ per kg of steam}} = \frac{244.43}{2408.04} = 0.1015 \text{ or } 10.15\% \]

(c) Using eqn. (9.24), Ideal steam consumption per kW-hour

\[ \frac{\text{heat equivalent of one kW-hour}}{\text{Work done per kg of steam}} = \frac{3,600}{244.43} = 14.73 \text{ kg per kW-hr} \]
Volume of superheated steam at 5.5 bar and 190°C

\[ v_{\text{sup}} = v_s \times \frac{T_{\text{sup}}}{T_{\text{sat}}} = 0.3427 \times \left( \frac{190 + 273}{155.48 + 273} \right) = 0.3703 \text{ m}^3/\text{kg} \]

\[ \therefore \text{Volume of steam supplied} = 14.73 \times 0.3703 = 5.4545 \text{ m}^3 \text{ per kW-hr.} \]

Problem-25: A steam engine is supplied with dry saturated steam at a pressure of 14 bar (14 MPa). Pressure at release is 0.6 bar (60 kPa) and exhaust takes place at 0.15 bar (15 kPa). Assuming isentropic expansion with constant volume conditions between release and commencement of exhaust, determine using steam tables only, (a) the work done per kg of steam (b) the mean effective pressure, and (c) the efficiency of the cycle.

Compare these values with those for Rankine cycle between the same pressure limits.

From steam tables:

<table>
<thead>
<tr>
<th>( p ) bar</th>
<th>( v_s ) m³/kg</th>
<th>( h ) kJ/kg</th>
<th>( L ) kJ/kg</th>
<th>( H ) kJ/kg</th>
<th>( \Phi_w ) kJ/kg K</th>
<th>( \Phi_s ) kJ/kg K</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>0.6</td>
<td>2.732</td>
<td>359.86</td>
<td>2,293.6</td>
<td>2,790</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>0.15</td>
<td>10.422</td>
<td>225.94</td>
<td>2,373.1</td>
<td>—</td>
<td>0.7549</td>
<td>8.0085</td>
</tr>
</tbody>
</table>

(a) Referring to fig. 9-23, and considering isentropic expansion 1-2 from 14 bar to 0.6 bar and then release at constant volume 2-4 from 0.6 bar to 0.15 bar (modified Rankine cycle):

Considering isentropic expansion 1 - 2:

\[ \Phi_{\text{S1}} = \Phi_{\text{w2}} + x_2 (\Phi_{\text{w2}} - \Phi_{\text{w2}}) \]

i.e. 6.4693 = 1.1453 + \( x_2 \) (7.532 - 1.1453)

\[ \therefore x_2 = 0.834 \]

From steam tables, \( H_1 = 2,790 \text{ kJ/kg} \);

\[ H_2 = h_2 + x_2 L_2 \]

\[ = 359.86 + 0.834 \times 2,293.6 = 2,272.72 \text{ kJ/kg} \]

Specific volume at 0.6 bar = 2.732 m³/kg (from steam tables).

\[ \therefore \text{Volume } v_2 = x_2 v_s = 0.834 \times 2.732 \text{ m}^3/\text{kg} = v_4 \]

Using eqn. (9.26a), work done per kg of steam in modified Rankine cycle

\[ = (H_1 - H_2) + [(P_2 - P_4) v_2] \]

\[ = (2790 - 2272.72) + [(60 - 15) \times 0.834 \times 2.732] \]

\[ = 620.35 \text{ kJ/kg of steam or } = 6,20,350 \text{ N.m/kg of steam} \]

(b) Mean effective pressure = \frac{\text{Work done per of steam in N.m}}{\text{Stroke volume in m}^3 \text{ per kg of steam (v4)}}

\[ = \frac{6,20,350}{0.834 \times 2.732} \text{ N/m}^2 \text{ or (Pa)} = \frac{6,20,350}{10^5 \times 0.834 \times 2.732} = 2.72 \text{ bar} \]

(\because 1 \text{ bar} = 10^5 \text{ Pa})
(c) Modified Rankine cycle efficiency
\[
\text{modified Rankine cycle efficiency} = \frac{\text{Work done per kg of steam in kJ}}{\text{Heat supplied per kg of steam in kJ}}
\]
\[
= \frac{620.35}{2,790 - 225.95} = 0.2419 \text{ or } 24.19\% 
\]

Considering Rankine cycle:

(a) Referring to fig. 9-23, and considering complete isentropic expansion 1-3 from 14 bar to 0.15 bar,
\[
\Phi_{s1} = \Phi_{s3} + x_3 (\Phi_{s3} - \Phi_{w3}) 
\]
i.e. \[6.4693 = 0.7549 + x_3 (8.0085 - 0.7549) \quad \therefore x_3 = 0.788 \]
Now, \[H_1 = 2,790 \text{ kJ/kg (steam tables), and} \]
\[H_3 = h_3 + x_3 L_3 = 225.94 + 0.788 \times 2,373.1 = 2,095.94 \text{ kJ/kg} \]
Work done \[= H_1 - H_3 = 2,790 - 2,095.94 = 694.06 \text{ kJ/kg}. \]
\[= 6,94,060 \text{ N.m/kg of steam.} \]

(b) Mean effective pressure = \[
\frac{6,94,060}{0.788 \times 10^{-022}} \text{ N.m}^2 \text{ or } \frac{6,94,060}{10^5 \times 0.788 \times 10^{-022}} = 0.879 \text{ bar (} \text{v} \text{1 bar} = 10^5 \text{ N/m}^2) \]

(c) Rankine cycle efficiency

\[
\text{Rankine cycle efficiency} = \frac{\text{Work done per kg of steam in kJ}}{\text{Heat supplied per kg of steam in kJ}}
\]
\[
= \frac{H_1 - H_3}{H_1 - h_3} = \frac{694.06}{2,790 - 225.94} \times 100 = 27.07\% 
\]

Problem-26: An engine is supplied with dry saturated steam at 8.5 bar. Expansion is carried out at constant entropy to a pressure of 1 bar (100 kPa) and the exhaust pressure is 0.15 bar (15 kPa). Sketch the ideal \(p-v\) and \(T-\Phi\) diagrams for the cycle and indicate there on the work done and the condition of steam at exhaust. Using the steam tables only, find the work done per kg of steam, the modified Rankine cycle efficiency, and the index of expansion.

Taking the Rankine cycle as standard, find the percentage loss of efficiency due to the incomplete expansion.

<table>
<thead>
<tr>
<th>(p) bar</th>
<th>(v_s) m(^3)/kg</th>
<th>(h) kJ/kg</th>
<th>(L) kJ/kg</th>
<th>(H) kJ/kg</th>
<th>(\Phi_w) kJ/kg K</th>
<th>(\Phi_s) kJ/kg K</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>0.227</td>
<td></td>
<td></td>
<td>2,771.6</td>
<td></td>
<td>6,6421</td>
</tr>
<tr>
<td>1</td>
<td>1.694</td>
<td>417.46</td>
<td>2,258</td>
<td>7,3594</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td></td>
<td>225.94</td>
<td>2,373.1</td>
<td></td>
<td>0.7549</td>
<td>8,0085</td>
</tr>
</tbody>
</table>

Considering modified Rankine cycle:

Referring to fig. 9-24(a) and considering isentropic expansion \(c-e\),

Entropy at \(c = \text{Entropy at } e\)
\[i.e. \ 6.6421 = 1.3026 + x_e (7.3594 - 1.3026) \quad \therefore x_e = 0.882 \text{ (dryness fraction at } e) \]
\[v_e = x_e v_{se} = 0.822 \times 1.694 = 1.5 \text{ m}^3/\text{kg}; \quad v_c = 0.227 \text{ m}^3/\text{kg (from steam tables)} \]
\[\therefore H_e = h_e + x_e L_e = 417.46 + 0.882 \times 2,258 = 2,409 \text{ kJ/kg} \]
Using eqn. (9.26a), Work done in modified Rankine cycle
\[ \text{Work done} = (H_c - H_d) + [(p_e - p_f) v_d] \]
\[ = (2,771.6 - 2,409) + [(100 - 15) 1.5] = 490.1 \text{ kJ/kg}. \]

Heat supplied = \( H_c - h_a = 2,771.6 - 225.94 = 2,545.66 \text{ kJ/kg} \).

Modified Rankine cycle efficiency,
\[ \text{Efficiency} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{490.1}{2,545.66} = 0.1925 \text{ or } 19.25\% \]

The isentropic expansion \( c-e \) of fig. 9-24 (b) is represented by equation \( pv^n = C \):

Now, \( p_e v_e^n = p_c v_c^n \) i.e. \( \frac{p_c}{p_e} = \left(\frac{v_e}{v_c}\right)^n \)
\[ \log \left(\frac{p_c}{p_e}\right) = \log \left(\frac{8.5}{1.5}\right) = \log \left(\frac{8.5}{0.227}\right) = 1.134 \]

Considering Rankine cycle:

Referring to fig. 9-24(a) and considering isentropic expansion \( c-d \).

Entropy at \( c \) = Entropy at \( d \)
\[ i.e. 6.6421 = 0.7549 + x_d (8.0085 - 0.7549) \therefore x_d = 0.812 \text{ (dryness fraction at } d) \]

Now, \( H_d = h_d + x_d L_d = 225.94 + 0.812 \times 2,373.1 = 2,152.9 \text{ kJ/kg} \)
\( H_c = 2,771.6 \text{ kJ/kg and } h_a = 225.94 \text{ kJ/kg (from steam tables)}. \)

Work done = \( H_c - H_d = 2,771.6 - 2,152.9 = 618.7 \text{ kJ/kg} \)

Heat supplied = \( H_c - h_a = 2,771.6 - 225.94 = 2,545.66 \text{ kJ/kg} \)

Rankine cycle efficiency = \[ \frac{\text{Work done in kJ/kg}}{\text{Heat supplied in kJ/kg}} = \frac{618.7}{2,545.66} = 0.243 \text{ or } 24.3\% \]
Percentage loss of efficiency due to incomplete expansion (in modified Rankine cycle)
\[\frac{24.3 - 19.25}{24.3} \times 100 = 20.78\%\]

9.12 Estimation of Missing Quantity

Mass of steam in the cylinder or the total steam present in the cylinder during expansion stroke consists of:

- mass of steam admitted per stroke, called the cylinder feed, and
- mass of steam contained in the clearance space before admission, called the cushion steam.

The mass of cylinder feed per stroke \(m_f\) can be determined experimentally by condensing exhaust steam in the condenser. From the measurement, cylinder feed per stroke,

\[
m_f = \frac{\text{total mass of exhaust steam condensed per minute}}{\text{number of strokes made in exhausting the steam per min}}
\]

or \[m_f = \frac{\text{cylinder feed per min}}{2 \times \text{r.p.m.}}\] (for double-acting steam engine) \[\text{.. (9.28)}\]

Cushion steam is the steam trapped in the clearance space during compression (after the closing of the exhaust valve), and it may be regarded as steam never exhausted. Assuming that the cushion steam is dry saturated after closing of exhaust valve, its mass \(m_c\) may be estimated by dividing the total volume (actual indicated volume) \(v_c\) of steam at some convenient point on the compression curve of the indicator diagram, say point \(c\) (fig. 9-25), by the specific volume of dry saturated steam \(v_{sc}\) at the corresponding pressure \(p_c\) obtained from steam tables. This assumption is justified as the cushion steam is small in quantity compared to the cylinder feed per stroke and only error of the second order is involved,

i.e. mass of cushion steam, \(m_c\)

\[
= \frac{\text{actual indicated volume at } c}{\text{volume of 1 kg of dry saturated steam}}
= \frac{v_c}{v_{sc}}
\]

The mass of steam present in the cylinder during expansion,

\[m_s = \text{cylinder feed per stroke, } m_f + \text{mass of cushion steam, } m_c\]

Now, consider any point \(b\) on the expansion curve (fig. 9-25) and read off from the diagram the pressure and volume of steam at point \(b\).

Let \(p_b\) = pressure of steam in bar at \(b\), and \(v_b\) = indicated volume of steam in \(m^3\) at \(b\).
From steam tables obtain the specific volume \( v_{sb} \) of dry saturated steam at pressure \( p_b \). Then, the volume, the steam at \( b \) would occupy if dry saturated = \( m_s \times v_{sb} \). This volume is represented by \( ab' \) to the volume scale of the indicator diagram. Thus, the point \( b' \) represents the volume which the steam at \( b \) would occupy if it was dry saturated.

Similarly, for every point on the expansion curve, a point corresponding to dry saturated condition can be obtained. A curve drawn through all these points is known as the saturation curve. The saturation curve shows at a glance the wetness (dryness fraction) of steam in the cylinder and volume of the missing quantity at any part of the expansion stroke.

Dryness fraction of steam at \( b \) = \( \frac{\text{actual indicated vol. at } b}{\text{dry steam volume}} = \frac{ab}{ab'} = \frac{vb}{m_s \times v_{sb}} \)  

The distance \( bb' \) (fig. 9-25) is the volume of missing quantity for the point \( b \).

It may be noticed from fig. 9-25 that the steam is wet at the beginning of the expansion stroke and becomes drier towards the end of the stroke. This is due to the fact that the high pressure steam at the commencement of the expansion stroke is hotter than the cylinder walls; this causes steam to condense. The steam pressure falls during the expansion stroke, and towards the end of the stroke the cylinder walls will be hotter than the steam; this will cause re-evaporation of condensed steam and the dryness fraction of steam will consequently increase (Refer illustrative Problem no.28).

Referring to indicator diagram of fig. 9-25, the point \( b' \) on saturation curve represents the volume, the steam would occupy, if it was dry saturated while the actual volume occupied is represented by the point \( b \) on the expansion curve. The horizontal distance between the saturation curve and expansion curve at any part of the stroke is known as missing quantity and is represented by \( bb' \) m\(^3\) at point \( b \). Alternatively, the missing quantity in kg per stroke may be regarded as the difference between the actual total mass of steam present in the cylinder during expansion stroke \( (m_f + m_c) \) and the indicated dry mass of steam at the point \( b \).

Referring to fig. 9-25,
Let \( p_b \) = pressure of steam in bar at \( b \).
\( v_b \) = volume of steam (actual indicated volume) in m\(^3\) at \( b \), and
\( v_{sb} \) = specific volume in m\(^3\)/kg at pressure \( p_b \) (from steam tables).

Then, indicated dry mass of steam at \( b \)
\[ = \frac{\text{actual indicated volume in } m^3 \text{ at } b}{\text{specific volume in } m^3 \text{ of } 1 \text{ kg of dry saturated steam at pressure } p_b} \]
\[ = \frac{v_b}{v_{sb}} \text{ kg} \]  

This mass is called the "indicated mass" because it is calculated by means of data obtained from a curve drawn by an indicator.

The dryness fraction of steam at \( b \) may also be expressed as
\[ \text{indicated dry mass of steam at } b \]
\[ = \frac{\text{total mass of steam present in the cylinder during expansion} = m_f + m_c}{\text{total mass of steam present in the cylinder during expansion} = m_f + m_c} \]
Missing quantity per stroke at $b$ = actual total mass of steam present in the cylinder during the expansion stroke ($m_f + m_c$) minus indicated dry mass of steam at $b$.

.. (9.33)

The area between the saturation curve and expansion curve represents the work lost due to missing quantity.

The missing quantity is mainly due to condensation (initially and during expansion) but a small amount will be due to leakage of steam past the piston rings and valves. The missing quantity may be of the order of 3% of the cylinder feed. This can be reduced by taking anti-condensation measures (methods) as discussed earlier in art. 9.7.

**Problem-27** : The following results were obtained by measurements taken on an indicator card from a double-acting steam engine:

<table>
<thead>
<tr>
<th>Volume $m^3$</th>
<th>Pressure bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1038</td>
<td>13</td>
</tr>
<tr>
<td>0.0285</td>
<td>4</td>
</tr>
</tbody>
</table>

The engine was running at 105 r.p.m. and the quantity of steam supplied per minute was 210 kg. Estimate the missing quantity in kg per minute and as a percentage of actual steam supplied, and the dryness fraction of steam at point $x$.

Volume of 1 kg of dry saturated steam at 4 bar is 0.4625 m$^3$ (from steam tables).

Assuming steam to be dry saturated at point $y$ (fig. 9-26) and using eqn. (9.29),

Mass of cushion steam = $m_c = \frac{\text{actual indicated volume at } y}{\text{vol. of 1 kg of dry saturated steam}} = \frac{0.0285}{0.4625} = 0.0616$ kg/stroke.

Mass of steam admitted per stroke (cylinder feed per stroke),

\[ m_f = \frac{\text{mass of steam supplied per min.}}{\text{no. of strokes (cycles) per min.}} = \frac{210}{2 \times 105} = 1 \text{ kg.} \]

:. The mass of steam present in the cylinder during expansion,

\[ m_s = m_f + m_c = 1 + 0.0616 = 1.0616 \text{ kg/stroke.} \]

Pressure at point $x$ is 13 bar and one kg of dry saturated steam at this pressure has a volume of 0.1513 m$^3$ (for steam tables).

Using eqn. (9.31), indicated dry mass of steam at point $x$

\[ = \frac{\text{actual indicated volume at } x}{\text{volume of 1 kg of dry saturated steam}} = \frac{0.1038}{0.1513} = 0.686 \text{ kg/stroke.} \]

Using eqn. (9.33), missing quantity at point $x$
= mass of steam present in the cylinder during expansion *minus* indicated dry mass of steam point $x$.

\[ = 1.0616 - 0.686 = 0.3756 \text{ kg/stroke}. \]

\[ \therefore \text{ Missing quantity/min. at point } x = 0.3756 \times 2 \times 10^5 = 78.88 \text{ kg per min.} \]

(engine being double-acting).

**Missing quantity at point** $x$ **as a percentage of actual mass of steam supplied**

\[ = \frac{78.88}{210} \times 100 = 37.56\% \]

Using eqn. (9.32), **dryness fraction of steam at point** $x$

\[ \text{indicated dry mass of steam at point } x \]

\[ \text{mass of steam present in the cylinder during expansion} \]

\[ = \frac{0.686}{1.0616} = 0.646 \text{ or } 64.6\% \text{ dry}. \]

**Problem-28** : The high-pressure cylinder of a double-acting steam engine running at 100 r.p.m. has an effective area of 250 cm$^2$ and a stroke of 60 cm and the clearance volume is 7 per cent of the stroke volume. During a trial the cylinder feed was 9 kg per minute. The indicator diagram showed that cut-off and release occurred at 30 and 90 per cent of the outward (expansion) stroke while the compression commenced at 70 per cent of the return (exhaust) stroke. The indicated pressures at cut-off, release and commencement of the compression are 14, 5.5 and 3 bar respectively. Find the quality of steam at cut-off and at release.

Referring to fig. 9-27, the swept volume,

\[ n_s = \frac{250}{10^4} \times \frac{60}{100} = 0.015 \text{ m}^3. \]

**Clearance volume**

\[ = \frac{7}{100} \times 0.015 = 0.00105 \text{ m}^3. \]

**Total volume (indicated) at A (at cut-off)**

\[ = \left( \frac{30}{100} \times 0.015 \right) + 0.00105 = 0.00555 \text{ m}^3. \]

**Total volume (indicated) at B (at release)**

\[ = \left( \frac{90}{100} \times 0.015 \right) + 0.00105 = 0.01455 \text{ m}^3. \]

**Total volume (indicated) at C (at commencement of compression)**

\[ = \left( \frac{100 - 70}{100} \times 0.015 \right) + 0.00105 \]

\[ = 0.00555 \text{ m}^3. \]

**Mass of steam admitted per stroke (cylinder feed per stroke),**

\[ m_f = \frac{9}{2 \times 100} = 0.045 \text{ kg} \] (engine being double-acting)
The specific volume of steam at 3 bar = 0.6058 m³/kg (from steam tables).

Using eqn. (9.29), the mass of cushion steam assumed to be dry saturated per stroke.

\[ m_c = \frac{\text{actual indicated volume at point C}}{\text{volume of 1 kg of dry saturated steam}} = \frac{0.00555}{0.6058} = 0.00916 \text{ kg per stroke.} \]

The mass of steam present in the cylinder during the expansion,

\[ m_s = m_r + m_c = 0.045 + 0.00916 = 0.05416 \text{ kg.} \]

At cut-off (Point A):

Total mass of steam present in the cylinder during expansion stroke = 0.05416 kg. The specific volume of steam at 14 bar = 0.1408 m³/kg (from steam tables). Hence, the steam if dry saturated at cut-off in the cylinder at 14 bar would have a volume of 0.05416 x 0.1408 = 0.00763 m³.

The total volume (actual indicated volume) of steam in the cylinder at point A (at cut-off) is found to be 0.00555 m³.

Using eqn. (9.30), quality (dryness fraction) of steam at point A (at cut-off)

\[ \frac{\text{actual indicated volume at A}}{\text{dry steam volume}} = \frac{0.00555}{0.00763} = 0.727 \text{ or 72.7\% dry.} \]

At release (point B):

Total mass of steam in the cylinder during expansion stroke = 0.05416 kg. The specific volume of steam at 5.5 bar = 0.3427 m³/kg (steam tables). Hence, the volume of steam if dry saturated at release in the cylinder at 5.5 bar = 0.05416 x 0.3427 = 0.01856 m³.

The total volume (actual indicated volume) of steam at release (at point B) in the cylinder is found to be 0.01455 m³.

Using eqn. (9.30), quality (dryness fraction) of steam at point B (at release)

\[ \frac{\text{actual indicated volume at B}}{\text{dry steam volume}} = \frac{0.01455}{0.01856} = 0.784 \text{ or 78.4\% dry.} \]

Problem-29: Determine the dryness fraction of steam and the missing quantity per hour at 0.7 of the expansion stroke, from the following data:

Diameter of cylinder, 40 cm; stroke, 60 cm; r.p.m., 100; cut-off at 50\% of the stroke; clearance volume, 8\% of swept volume; steam condensed per min., 40 kg; pressure of steam at 0.7 of the expansion stroke, 5 bar; pressure of steam at 0.8 of the return stroke on the compression curve, 1.4 bar. State any assumptions made.

Referring to fig. 9-28, the swept volume,

\[ v = \frac{\pi}{4} \left( \frac{40}{100} \right)^2 \times \frac{60}{100} = 0.0754 \text{ m}^3 \]
The clearance volume = 0.08 \times 0.08 \times 0.0754 = 0.00603 \, m^3.

Total volume (actual indicated volume) of steam at A, which is at 0.7 of the expansion stroke,

= 0.7 \times \text{swept volume} + \text{clearance volume}

= (0.7 \times 0.0754) + 0.00603 = 0.05881 \, m^3.

Total volume (actual indicated volume) of steam at B, which is at 0.8 of the return stroke on the compression curve

= (1 - 0.8) \times \text{swept vol.} + \text{clearance vol.}

= (0.2 \times 0.0754) + 0.00603 = 0.02111 \, m^3.

The specific volume of dry saturated steam at 1.4 bar = 1.2366 \, m^3/kg (from steam tables).

Using eqn. (9.29), the mass of cushion steam assumed to be dry saturated per stroke,

\[ m_c = \frac{\text{actual indicated volume at B}}{\text{volume of 1 kg of dry saturated steam}} = \frac{0.02111}{1.2366} = 0.01707 \, \text{kg}. \]

Using eqn. (9.28), mass of steam admitted per stroke (cylinder feed per stroke),

\[ m_f = \frac{\text{mass of steam condensed per minute}}{\text{no. of strokes (cycles) per minute}} = \frac{40}{2 \times 100} = 0.2 \, \text{kg} \, \text{(assuming double-acting steam engine)} \]

The mass of steam present in the cylinder during expansion,

\[ m_s = m_c + m_f = 0.01707 + 0.2 = 0.21707 \, \text{kg}. \]

Considering point A on the indicator diagram i.e. at 0.7 of the expansion stroke:

Pressure at A is 5 bar and 1 kg of steam dry saturated at this pressure has a volume of 0.3749 \, m^3 (from steam tables).

Hence, if steam dry and saturated, the steam at point A would have a volume of

\[ 0.21707 \times 0.3749 = 0.08138 \, \text{m}^3. \]

But, from the indicator diagram the actual indicated volume of steam at point A is 0.05881 \, m^3.

Using eqn. (9.30), Dryness fraction of steam at A

\[ = \frac{\text{actual indicated volume at A}}{\text{dry steam volume}} = \frac{0.05881}{0.08138} = 0.7226 \, \text{or} \, 72.26\% \, \text{dry} \]

Using eqn. (9.31), Indicated dry mass of steam at A

\[ = \frac{\text{actual indicated volume at A}}{\text{volume of 1 kg of dry saturated steam}} = \frac{0.05881}{0.3749} = 0.15686 \, \text{kg/stroke}. \]

Using eqn. (9.33), Missing quantity per stroke at A

\[ = \text{total mass of steam present in the cylinder during expansion minus indicated dry mass of steam at A} = 0.21707 - 0.15686 = 0.06021 \, \text{kg per stroke}, \]

\[ \therefore \, \text{Missing quantity at A per min} = 0.06021(2 \times 100) = 12.042 \, \text{kg/min}. \]

\[ = 722.52 \, \text{kg/hr}. \]
Alternatively, using eqn. (9.32),
\[
\text{Dryness fraction of steam at A (i.e. at 0.7 of the expansion stroke)} = \frac{\text{indicated dry mass of steam at A}}{\text{mass of steam present in the cylinder during expansion}}
\]

\[
= \frac{0.15686}{0.21707} = 0.7226 \text{ or } 77.26\% \text{ (same as before)}
\]

Problem-30: A double-acting steam engine, having a diameter of 70 cm and stroke 120 cm runs at 75 r.p.m. Clearance volume is 7% of the swept volume. The cylinder feed is 1.1 kg per stroke. From the indicator card, the following readings were obtained:

(i) At a point on compression curve - pressure 3.5 bar and volume 0.1133 m³.
(ii) Cut-off at 1/2 stroke - pressure and volume just after cut-off are 11 bar and 0.2 m³ respectively.
(iii) At half stroke - pressure 8 bar (on expansion curve).
(iv) At release - pressure 4.5 bar, volume 0.485 m³.

Estimate the dryness fraction and missing quantity in kg per hour at (a) cut-off, (b) half-stroke, and (c) release.

Referring to fig. 9-29, actual indicated volume at \(D = 0.1133 \text{ m}^3\) (given).

1 kg of dry saturated steam at 3.5 bar has a volume of 0.5243 m³ (from steam tables).

Using eqn. (9.29), mass of cushion steam assumed to be dry saturated per stroke,

\[
m_c = \frac{\text{Actual indicated volume at } D}{\text{volume of 1 kg of dry saturated steam}}
\]

\[
= \frac{0.1133}{0.5243} = 0.216 \text{ kg/stroke.}
\]

The cylinder feed per stroke,

\[
m_f = 1.1 \text{ kg (given)}
\]

The mass of steam present in the cylinder during expansion,

\[
m_s = m_f + m_c = 1.1 + 0.216 = 1.316 \text{ kg/stroke.}
\]

(a) At cut-off (Point A):

Volume of 1 kg of steam dry saturated at 11 bar = 0.1775 m³ (from steam tables).

Hence, if steam is dry saturated, the steam at this pressure would have a volume of

\[
1.316 \times 0.1775 = 0.2336 \text{ m}^3
\]

Actual indicated volume of steam in the cylinder at A = 0.2 m³ (given).

Using eqn. (9.30), dryness fraction of steam at A (at cut-off)

\[
= \frac{\text{actual indicated volume at A}}{\text{dry steam volume}} = \frac{0.2}{0.2336} = 0.856 \text{ or } 85.6\% \text{ dry.}
\]

Using eqn. (9.31), indicated dry mass of steam at A

\[
HEI - 19
\]
actual indicated volume at A

volume of 1 kg of dry saturated steam = \( \frac{0.2}{0.1775} = 1.1268 \) kg/ stroke.

Using eqn. (9.33), Missing quantity at A (at cut-off)

\( 1.316 - 1.1268 = 0.1892 \) kg/ stroke or \( 0.1892 \times (2 \times 75) \times 60 = 1,703 \) kg/hr.

(b) At half-stroke (Point B) :

Swept volume,

\[ V = \pi \left( \frac{70}{100} \right) \times \frac{120}{100} = 0.462 \ m^3. \]

Clearance volume = \( 0.07 \times 0.462 = 0.03234 \ m^3. \)

\[ \therefore \text{Total indicated volume of steam at half stroke} = (0.462 \times 0.5) + 0.03234 = 0.26334 \ m^3. \]

1 kg of dry saturated steam at 8 bar has a volume of 0.2404 m\(^3\) (from steam tables). Hence, if steam is dry saturated, the steam at this pressure would have a volume of 1.316 \( \times 0.2404 = 0.3164 \) m\(^3\).

Using eqn. (9.30), Dryness fraction of steam at half-stroke (at point B)

\[ \text{actual indicated volume at B \over \text{dry steam volume}} = \frac{0.26334}{0.3164} = 0.832 \text{ or } 83.2\% \text{ dry.} \]

Using eqn. (9.31), Indicated dry mass of steam at B

\[ \text{actual indicated volume at B \over \text{volume of 1 kg of dry saturated steam}} = \frac{0.26334}{0.2404} = 1.095 \text{ kg/ stroke.} \]

\[ \therefore \text{Missing quantity at B (half-stroke)} = 1.316 - 1.095 = 0.221 \text{ kg/ stroke} \]

\[ = 0.221 \times (2 \times 75) \times 60 = 1,989 \text{ kg/hr}. \]

(c) At release (Point C) :

One kg of dry saturated steam at 4.5 bar has a volume of 0.414 m\(^3\) (from steam tables). Hence, if steam is dry saturated, the steam at this pressure would have a volume of 1.316 \( \times 0.414 = 0.5448 \) m\(^3\).

Indicated volume of steam in the cylinder at C = 0.485 m\(^3\) (given)

Dryness fraction of steam at C (at release)

\[ \text{actual indicated volume at C \over \text{dry steam volume}} = \frac{0.485}{0.5448} = 0.89 \text{ i.e. } 89\% \text{ dry.} \]

Indicated dry mass of steam at C (at release)

\[ \text{actual indicated volume at C \over \text{volume of 1 kg of dry saturated steam}} = \frac{0.485}{0.414} = 1.1715 \text{ kg/ stroke.} \]

Missing quantity at C (at release) = 1.316 - 1.1715 = 0.1445 kg/ stroke.

\[ = 0.1445 \times (2 \times 75) \times 60 = 1,300.5 \text{ kg/hr}. \]

9.13 Methods of Governing Steam Engines

The function of a governor is to maintain the speed of the engine constant by either controlling the quantity of steam or the pressure of the steam supplied to the engine according to the load on the engine.

There are two methods of maintaining the speed of steam engines constant by the action of the governor:
By varying the point of cut-off (altering the period of admission of steam to the engine cylinder). This method of governing is known as cut-off governing.

By throttling the entering steam. This method is known as throttle governing.

In cut-off governing, the volume of steam supplied to the engine is altered by changing the point of cut-off by a special slide valve working under the control of the governor. In the throttle governing, the supply of steam is throttled to a lower pressure by partial closure of throttle valve under the control of a governor. The indicator diagram showing the effect of cut-off governing and throttle governing at various loads are shown in fig. 9-30.

![Diagram](image)

Fig. 9-30. Effects of cut-off and throttle governing on P–V diagram at various loads.

9.13.1 Willan's Law: During a test on steam engine, the indicated power is varied by altering the load, and the rate of steam consumption is obtained from the mass of steam condensed by the condenser.

Figure 9-31 shows graphs of steam consumption in kg/hr. plotted against load in terms of indicated power in kW for both throttle governing and cut-off governing. It will be noticed that the graph is a straight line in case of throttle governing and a smooth curve in case of cut-off governing. The straight line graph in case of throttle governing indicates that steam consumption per hour is directly proportional to the indicated power. This is known as Willan's law, and holds good only for a throttle-governed engine running at constant speed. In general the straight line can be obtained for the steam consumption graph only if the ratio of expansion or cut-off is constant; this condition is not fulfilled in a cut-off governed engine.

The intercepts of the steam consumption lines on the vertical axis at no load, represent the steam consumption required for idle (no load) running of the engine, i.e. for overcoming frictional power of the engine.
For an engine governed by throttle governing, Willan's line is a straight line and may therefore be expressed in the form of

\[ S = a + b I \]

where \( S \) = steam consumption in kg/hr.,
\( a \) = steam consumption at no load in kg/hr. (constant),
\( b \) = slope of Willan's line (constant), and
\( I \) = load on the engine in terms of indicated power in kW.

**Problem-31**: A throttled governed engine running at constant speed uses 310 kg of steam per hour when developing indicated power of 15 kW and 574 kg of steam per hour when developing indicated power of 37 kW. Estimate the steam consumption in kg/hr. when developing indicated power of 22 kW.

For a throttled governed engine, running at constant speed, steam consumption in kg/hr. (\( S \)) can be expressed in terms of indicated power (\( I \)) in kW as

\[ S = a + b I \]

where \( a \) and \( b \) are constants.

Substituting the values of steam consumption for the two loads in the above equation we have,

\[ 310 = a + b \times 15 \quad \ldots \text{(a)} \]
\[ 574 = a + b \times 37 \quad \ldots \text{(b)} \]

Subtracting (a) from (b), we have,

\[ 264 = 22b \quad \therefore b = 12 \]

Substituting value of \( b \) in equation (a), we have,

\[ 310 = a + 12 \times 15 \quad \therefore a = 150 \]

Now, steam consumption when developing indicated power of 22 kW,

\[ S = a + b \times 22 \]
\[ = 150 + 12 \times 22 = 414 \text{ kg/hr.} \]

**Problem-32**: A small steam engine uses 115 kg of steam per hour when developing indicated power of 4 kW. The speed is kept constant by throttle governor and the steam consumption is 195 kg per hour when developing indicated power of 9.34 kW. Calculate the indicated thermal efficiency of the engine at the same speed when developing indicated power of 7.35 kW. The steam supplied is at a pressure of 4 bar and 0.95 dry. The temperature of condensate is 44°C.

Compare the indicated thermal efficiency with the Rankine cycle efficiency if the condenser pressure is 0.2 bar.

For a throttle governed steam engine running at constant speed, steam consumption in kg/hr (\( S \)) can be expressed in terms of indicated power (\( I \)) in kW as

\[ S = a + b I \]

where \( a \) and \( b \) are constants.

Substituting the values of the steam consumption for two loads in the above equation, we have,

\[ 115 = a + b \times 4.0 \quad \ldots \text{(a)} \]
\[ 195 = a + b \times 9.34 \quad \ldots \text{(b)} \]
Subtracting (a) from (b), we have, 80 = \(5.34\ a\) \(:= \ b = 14.98\)

Substituting value of \(b\) in eqn. (b), we have, 195 = \(a + 14.98 \times 9.34\) \(:= \ a = 55\)

Using eqn. (9.34), steam consumption when developing 7.35 kW indicated power,
\[S = a + b \times 7.35 = 55 + 14.98 \times 7.35 = 165\] kg per hour.

At 4 bar, \(h = 604.74\) kJ/kg. \(L = 2.133.8\) kJ/kg (from steam tables).

Now, \(H_I = h_I + x_I L_I = 604.75 + 0.95 \times 2.133.8 = 2.631.85\) kJ/kg.

Heat remaining in condensate, \(h_2 = 4.187 (t_c - 0) = 4.187 (44 - 0) = 184\) kJ/kg.

Heat supplied/hr. = \(S (H_I^* - h_2) = 165 (2.631.85 - 184)\) kJ/hr.

Indicated thermal efficiency = \(\frac{\text{Heat equivalent of indicated power in kJ per hr.}}{\text{Heat supplied in kJ per hr.}}\)
\[\frac{K \times 3.600}{S (H_I^* - h_2)} = \frac{7.35 \times 3.600}{165 (2.631.85 - 184)} = 0.0655\] or 6.55%

Rankine cycle: Rankine heat drop (work done) from 4 bar and 0.95 dry to 0.2 bar = \(H_I - H_2 = 445\) kJ/kg (from \(H - \Phi\) chart).

At 0.2 bar, \(h_2 = 251.4\) kJ/kg (from steam tables)

(Where \(h_2 = \text{Enthalpy of 1 kg of water of exhaust steam}.)

Heat supplied per kg of steam = \(H_I - h_2 = 2.631.85 - 251.4 = 2.380.45\) kJ/kg.

Rankine cycle efficiency = \(\frac{\text{heat drop (work done) per kg of steam}}{\text{heat supplied per kg of steam}}\)
\[
\frac{H_I - H_2}{H_I - h_2} = \frac{445}{2.380.45} = 0.1869\] or 18.69%

Efficiency ratio = \(\frac{\text{Indicated thermal efficiency}}{\text{Rankine cycle efficiency}}\)
\[
\frac{6.55}{18.69} \times 100 = 35.00\%
\]

Problem-33: A throttle governed steam turbine, running at constant speed develops indicated power of 926.5 kW while consuming 7,430 kg of steam per hour. When the steam consumption is 10,000 kg per hour it develops indicated power of 1,323 kW. If the steam supply is at 24 bar with 100°C of superheat, find (without using Mollier chart) the efficiency ratio for the turbine while developing indicated power of 1,103 kW. The back pressure is 0.1 bar. Take \(K_p\) of superheated steam as 2.35 kJ/kg K.

For a throttle governed steam turbine running at constant speed, steam consumption in kg/hour (S) can be expressed in terms of indicated power (I) in kW as
\[S = a + b I\]
where \(a\) and \(b\) are constants

Substituting the values of the steam consumption for two loads in the above equation
\[7,430 = a + b \times 926.5\]
\[10,000 = a + b \times 1,323\]
Subtracting (a) from (b), we have, 2,570 = 396.5 \(b\) \(:= \ b = 6.48\)

Substituting value of \(b\) in eqn. (a), we get, 7,430 = \(a + 6.48 \times 926.5\) \(:= \ a = 1,426.3\)

Using eqn. (9.34), steam consumption when developing indicated power of 1,103 kW.
\[S = a + b \times 1,103 = 1,426.3 + 6.48 \times 1,103 = 8,574\] kg/hr.
At 24 bar, \( H_s = 2,802.6 \text{ kJ/kg} \) (from steam tables).

Now, \( H_1 = H_s - K_p (t_{sup} - t_{sat}) = 2,802.6 + 2.35 (100) = 3,037.6 \text{ kJ/kg} \).

At 0.1 bar, \( h_2 = 191.83 \text{ kJ/kg} \) (from steam tables).

Heat supplied/hour = \( S (H_1 - h_2) = 8,574 \) (3,037.6 - 191.83) kJ per hour

Now, indicated thermal efficiency \( \eta_1 = \frac{\text{heat equivalent of indicated power in kJ per hr}}{\text{heat supplied in kJ/hr}} \)

\[
= \frac{kW \times 3,600}{S(H_1 - h_2)} = \frac{1,103 \times 3,600}{8,574 \times (3,037.6 - 191.83)} = 0.1627 \text{ or } 16.27\%.
\]

Rankine cycle: From steam tables, at 24 bar, \( \Phi_s = 6.2729 \text{ kJ/kg K}, \ t_s = 221.83^\circ \text{C}, \)
and at 0.1 bar \( \Phi_w = 0.6493 \text{ kJ/kg K}, \ \Phi_s = 8.1502 \text{ kJ/kg K}, \ h = 191.83 \text{ kJ/kg}, \ L = 2.5847 \text{ kJ/kg} \).

Now, entropy before expansion, \( \Phi_1 = \text{entropy after expansion, } \Phi_2 \)

i.e. \( \Phi_{s1} + K_p \log_e \frac{T_{sup}}{T_{sat}} = \Phi_{w2} + x_2 (\Phi_s2 - \Phi_w2) \)

i.e. \( 6.2729 + 2.35 \log_e \left( \frac{100 + 221.83 + 273}{221.83 + 273} \right) = 0.6493 + x_2 (8.1502 - 0.6493) \)

\[
\therefore \ x_2 = 0.8073
\]

\( H_2 = h_2 + x_2L_2 = 191.83 + 0.8073 \times 2.5847 = 2,278.5 \text{ kJ/kg}. \)

Rankine cycle efficiency, \( = \frac{\text{work done per kg of steam}}{\text{heat supplied per kg of steam}} \)

\[
= \frac{H_1 - H_2}{H_1 - h_2} = \frac{3,037.6 - 2,278.5}{3,037.6 - 191.83} = 0.267 \text{ or } 26.7\%.
\]

\[\therefore \text{Efficiency ratio} = \frac{\text{Indicated thermal efficiency}}{\text{Rankine cycle efficiency}} = \frac{16.27}{26.7} \times 100 = 61\%.
\]

Problem-34: In a steam engine plant the boiler supplies the engine dry saturated steam at 14 bar and the condenser pressure is 0.3 bar. Calculate the Rankine cycle efficiency of the engine. Using the steam tables only and selecting your own values for the relative efficiency, mechanical efficiency of the engine, and boiler thermal efficiency, estimate the probable overall efficiency of the steam engine plant from coal to brake.

From steam tables at 14 bar, \( H_s = 2,790 \text{ kJ/kg}, \ \Phi_s = 6.4693 \text{ kJ/kg K}, \) and at 0.3 bar, \( h = 289.23 \text{ kJ/kg}, \ L = 2.3361 \text{ kJ/kg}, \ \Phi_w = 0.9439 \text{ kJ/kg K}, \)

\( \Phi_s = 7.7686 \text{ kJ/kg K}. \)

Entropy before expansion, \( \Phi_1 = \text{Entropy after expansion, } \Phi_2 \)

i.e. \( \Phi_{s1} = \Phi_{w2} + x_2 (\Phi_s2 - \Phi_w2) \)

i.e. \( 6.4693 = 0.9439 + x_2 (7.7686 - 0.9439) \)

\[\therefore \ x_2 = 0.81 \]

Now, \( H_1 = 2,790 \text{ kJ/kg} \) (from steam tables),

\( h_2 = 289.23 \text{ kJ/kg} \) (from steam tables).

\[\therefore H_2 = h_2 + x_2L_2 = 289.23 + 0.81 \times 2.3361 = 2,181.7 \text{ kJ/kg}. \]

Work done = \( H_1 - H_2 = 2,790 - 2,181.7 = 608.3 \text{ kJ/kg}. \)
Heat supplied = \( H_1 - h_2 = 2,790 - 289.23 = 2,500.77 \) kJ/kg.

Rankine cycle efficiency = \( \frac{\text{work done per kg of steam}}{\text{heat supplied per kg of steam}} \)

\[ \frac{H_1 - h_2}{H_1 - h_2} = \frac{608.3}{2,500.77} = 0.2432 \text{ or } 24.32\% \]

Taking relative efficiency as 60%, we have, \( 0.6 = \frac{\text{Indicated thermal efficiency}}{\text{Rankine cycle efficiency}} \)

\[ \therefore \text{Indicated thermal efficiency, } \eta_i = 0.6 \times 0.2432 = 0.146 \text{ or } 14.6\% \]

Now, brake thermal efficiency, \( \eta_b = \text{indicated thermal efficiency } \times \text{mechanical efficiency} \)

Taking mechanical efficiency, \( \eta_m = 80\% \),

\[ \text{Brake thermal efficiency, } \eta_b = 0.146 \times 0.8 = 0.1168 \text{ or } 11.68\% \]

Using eqn. (9.17), Brake thermal efficiency, \( \eta_b = \frac{\text{Brake power in kW}}{\text{mass of steam/sec. } \times (H_1 - h_2)} \)

Assuming that heat of condensate \( h_2 \) is not returned to hot-well, i.e. neglecting heat of condensate, \( h_2 \), we have,

\[ \text{Brake thermal efficiency, } \eta_b = \frac{\text{brake power in kW}}{\text{mass of steam/sec. } \times \text{enthalpy of 1 kg of steam}} \]

\[ \text{Boiler thermal efficiency} = \frac{\text{mass of steam per sec. } \times \text{enthalpy of 1 kg of steam}}{\text{mass of coal/sec. } \times \text{C.V. of coal}} \]

\[ \therefore \text{Brake thermal efficiency of the engine } \times \text{boiler thermal efficiency} \]

\[ = \left[ \frac{\text{brake power in kW}}{\text{mass of steam/sec. } \times \text{enthalpy of 1 kg of steam}} \right] \times \left[ \frac{\text{mass of steam/sec. } \times \text{enthalpy of 1 kg of steam}}{\text{mass of coal/sec. } \times \text{C.V. of coal}} \right] \]

\[ = \frac{\text{brake power in kW}}{\text{mass of coal/sec. } \times \text{C.V. of coal}} \]

Now, over-all efficiency of the steam plant from coal to brake

\[ = \frac{\text{brake power in kW}}{\text{mass of coal/sec. } \times \text{C.V. of coal}} \]

\[ = \text{brake thermal efficiency of the engine } \times \text{boiler thermal efficiency} \]

Taking boiler thermal efficiency as 75%,

Overall efficiency of the plant from coal to brake = \( 0.1168 \times 0.75 \)

\[ = 0.0876 \text{ or } 8.76\%. \]

1. (a) What is a heat engine?
   (b) Why is steam engine known as a prime mover?
   (c) Distinguish between external combustion and internal combustion engines.

2. Describe briefly the function of the following parts of steam engine:
   (a) Eccentric, (b) Piston, (c) Stuffing box and gland, (d) Crosshead, (e) Crank, (f) Flywheel, (g) Connecting rod, (h) Gudgeon pin, and (i) D-slide valve.
3. Explain the following terms as applied to steam engines:

4. (a) Show that hypothetical m.e.p. of steam engine may be expressed as

   \[ \text{Hypothetical m.e.p.} = \frac{P_i}{r} \left[ 1 + \log_e N \right] - P_b \]

   State clearly the meaning of each symbol.

   (b) Steam is admitted to the cylinder of steam engine at a pressure of 735 kPa and cut-off takes place at 0.4 of the stroke. The back pressure is 29.5 kPa. Calculate the hypothetical mean effective pressure on the piston during the stroke.

5. (a) Explain the following terms as applied to steam engines:
   (i) Mean effective pressure, (ii) Back pressure, (iii) Brake power, (iv) Indicated power, and (v) Diagram factor.

   (b) Sketch and explain the working of any one type of steam engine indicator you know.

6. (a) Explain why steam engines are provided with steam jacket.

   (b) Write short notes on the following and explain them with the help of sketches wherever necessary:
   (i) Indicator, (ii) Mechanical efficiency, (iii) Indicated thermal efficiency, (iv) Brake thermal efficiency, and (v) Overall efficiency of a steam engine plant.

7. What do you understand by the term “diagram factor”? Give its average value.
   A single-cylinder, double-acting steam engine of 20 cm diameter and 40 cm stroke is supplied with steam at 8.5 bar and exhausts at 0.15 bar. Cut-off takes place at \( \frac{1}{3} \) rd of the stroke and the engine runs at 120 r.p.m. Using the diagram factor of 0.7, estimate the actual m.e.p. and indicated power of the engine.

8. A double-acting, simple steam engine, having a slide valve, has a bore and stroke, 30 cm and 38 cm respectively. It runs at an average speed 200 r.p.m. and is supplied with steam at 7 bar. The vacuum gauge of its condenser indicates 650 mm of Hg, while barometer indicates 750 mm of Hg. If the valve is set to give a constant cut-off at 40% of the stroke and a diagram factor of 0.8, find indicated power of the engine.

9. Obtain an expression for the hypothetical mean effective pressure in a steam engine cylinder, neglecting the clearance volume, stating clearly the assumptions you make.
   A single-cylinder, double-acting, steam engine is to develop indicated power of 185 kW at 120 r.p.m., the engine stroke being twice the diameter of the cylinder. Steam supply is at 10 bar and the exhaust pressure is 0.2 bar. Cut-off takes place at \( \frac{1}{4} \) th of the stroke. Using a diagram factor of 0.85, calculate the required diameter of the cylinder and piston stroke.

10. (a) Explain what is meant by the mean effective pressure of a steam engine and show how its value is obtained from an indicator card.

    (b) A single-cylinder, double-acting steam engine has a bore and stroke, 25 cm and 35 cm respectively. Cut-off is at \( \frac{1}{4} \) rd of stroke. The initial steam pressure is 10 bar, and the exhaust pressure is 1.1 bar and the engine is found to develop indicated power of 30 KW at 120 r.p.m. Calculate the diagram factor of the engine.

11. (a) Derive an expression for the hypothetical mean effective pressure in a steam engine cylinder, in terms of steam supply pressure \( P_i \), back pressure \( P_b \) and expansion ratio \( r \).

    (b) A single-cylinder, double-acting steam engine having cylinder diameter 20 cm and stroke 30 cm, is working with 40% cut-off and running at 200 r.p.m. The engine is supplied with dry saturated steam at 8.5 bar. The back pressure is 0.25 bar. Calculate the brake power of the engine, assuming the mechanical efficiency of 80% and a diagram factor of 0.75.

12. What is meant by the term clearance?
    A steam engine has a cylinder diameter 30 cm and stroke 40 cm and has a clearance volume of 7.5 per cent of stroke volume. If the cut-off takes place at \( \frac{1}{4} \) th of the stroke, what is the true or real ratio of expansion?
13. Find the diameter of the cylinder of a single-cylinder, double-acting steam engine developing indicated power of 75 kW at a piston speed of 180 metres/minute and operating under the following conditions:
- Initial pressure, 10 bar
- Back pressure, 1.5 bar
- Cut-off at 1/6th of stroke
- Clearance volume, 10% of the piston displacement

A diagram factor of 0.85 may be assumed and effect of compression be neglected.

\[ 29.17 \text{ cm} \]

14. A single-cylinder, double-acting steam engine has a cylinder diameter of 75 cm and a stroke of 120 cm. Dry saturated steam is admitted to the engine at a pressure of 6.5 bar and cut-off takes place at 50% of the stroke. The back pressure is 1.1 bar. The crank is rotating at 300 r.p.m. Neglecting clearance and assuming a diagram factor of 0.8, calculate the m.e.p., indicated power and theoretical steam consumption in kg per kW per hour based on indicated power.

\[ 3.52 \text{ bar; 1,866 kW; 17.47 kg} \]

15. A single-cylinder, double-acting steam engine has the cylinder diameter 30 cm and stroke 50 cm. Steam is admitted at a pressure of 7 bar and cut-off is at 0.35 stroke. The back pressure is 1.1 bar. The brake power and mechanical efficiency of the engine at a speed of 150 r.p.m. are 45 kW and 81% respectively. Find the diagram factor of the engine.

\[ 0.801 \]

16. A double-acting, simple steam engine receives steam at a pressure of 7 bar. The cut-off takes place at 1/2 stroke. Find the indicated power developed by the engine at 210 r.p.m. The diameter of the piston and the piston rod are 20 cm and 6 cm respectively and the stroke is 50 cm. Assume a diagram factor of 0.75 for both ends. Back pressure may be taken as 1.2 bar.

\[ 37.215 \text{ kW} \]

17. The following observations were made during a test of a single-cylinder, double-acting steam engine:
- Area of indicator diagram: 14.5 cm²
- Length of indicator diagram: 10 cm
- Indicator spring scale: 200 kPa/cm
- Cylinder diameter: 17 cm
- Stroke: 30 cm
- R.p.m.: 250

Calculate the m.e.p. and indicator power of the engine.

\[ 290 \text{ kPa; 16.455 kW} \]

18. A single-cylinder, double-acting steam engine of 30 cm cylinder diameter and 36 cm piston stroke, is supplied with steam at a pressure of 875 kPa. Cut-off takes place at 40 per cent of the stroke and the engine runs at 3 r.p.s. (revolutions per second). The back pressure is 14.7 kPa. Using the diagram factor of 0.8, estimate the indicated m.e.p. and indicated power of the engine.

\[ 845.68 \text{ kPa; 64.58 kW} \]

19. During a test on a double-acting, single-cylinder steam engine running at 140 r.p.m., the indicated power was 81 kW, cylinder diameter 30 cm, stroke 45 cm, cut-off at 1/3rd stroke, initial steam pressure 1,100 kPa, back pressure 40 kPa. Find the diagram factor.

\[ 0.748 \]

20. In a trial on a single-cylinder, double-acting vertical steam engine, the following observations were made:
- Cylinder diameter, 17 cm
- Piston rod diameter, 5 cm
- Stroke, 30 cm
- R.p.m., 240
- Length of both indicator diagrams, 10 cm
- Area of top end indicator diagram, 14.5 cm²
- Area of bottom end indicator diagram, 13 cm²
- Indicator spring scale, 250 kPa/cm
- Circumference of brake wheel, 4.25 metres
- Circumferences of brake rope, 3 cm
- Dead load on the brake, 900 newtons
- Reading of spring balance, 75 newtons

Calculate the indicated power, brake power and mechanical efficiency of the engine.

\[ 17.96 \text{ kW; 14.124 kW; 78.64%} \]

21. Explain the term "Indicated thermal efficiency" as applied to a steam engine.

A single-cylinder, double-acting steam engine of 28 cm bore and 45 cm stroke, works between a supply pressure of 10 bar and back pressure of 0.15 bar. Assuming diagram factor of 0.7 and neglecting the clearance volume, estimate the indicated power developed at 180 r.p.m., if the cut-off occurs at 1/3rd stroke.

If the above engine consumes 900 kg of dry saturated steam per hour, determine the indicated thermal efficiency of the engine.

\[ 79.66 \text{ kW; 12.48%} \]

22. A double-acting, single-cylinder steam engine with cylinder 35 cm diameter and 53 cm stroke, is to develop indicated power of 147.8 kW at 200 r.p.m. with a cut-off at 0.4 stroke. Determine the admission pressure of steam if the exhaust pressure is 25 kPa and the diagram factor is 0.85.

If the above engine consumes 1,980 kg of steam per hour and the steam at admission is dry saturated, determine the indicated thermal efficiency of the engine.

\[ 700 \text{ kPa; 10.79%} \]
23. Explain the term “brake thermal efficiency” as applied to a steam engine.

The following results refer to a test on a double-acting, simple steam engine with a cylinder 22 cm in diameter and stroke 30 cm: Engine speed, 2 r.p.s; Mean effective pressure 245 kPa; Brake wheel diameter, 1.4 metres; Net load on the brake wheel, 981 newtons; Steam supplied at a pressure of 7 bar, dry saturated; Exhaust pressure, 1 bar; Steam used per hour, 180 kg. Determine: (i) the indicated power, (ii) the brake power (power output), (iii) the mechanical efficiency, (iv) the brake thermal efficiency, and (v) the indicated thermal efficiency of the engine.

[(i) 11-17 kW; (ii) 8-63 kW; (iii) 77-26%; (iv) 7-36%; (v) 9-52%]

24. Define the terms “diagram factor” and “indicated thermal efficiency” as applied to the reciprocating steam engines and explain the practical use of these terms.

The admission pressure of steam to a single-cylinder, double-acting steam engine is 10 bar. Cut-off takes place at 0-4 of the stroke and back pressure is 1-1 bar. The cylinder diameter is 35 cm and the stroke is 53 cm. Taking a diagram factor of 0-75, calculate the m.e.p. Assuming mechanical efficiency of 80 per cent, estimate the brake power of the engine at 200 r.p.m. Neglect clearance.

If the indicated thermal efficiency of the above engine is 14%, calculate the steam supplied in kg per kW per hour based on indicated power. The engine is supplied with dry saturated steam.

[4.924 bar; 133.912 kW; 10.946 kg]

25. The following results refer to a trial on a double-acting, simple steam engine with a cylinder 22 cm in diameter and stroke 30 cm, brake wheel 1-4 metres in diameter: dead load on the brake, 1,177 newtons; spring balance reading, 196 newtons; engine speed, 140 r.p.m.; mean effective pressure, 2-5 bar; steam supplied is dry saturated at a pressure of 5 bar; exhaust pressure, 1-1 bar; steam used per hour, 180 kg. Determine the mechanical efficiency, brake thermal efficiency and indicated thermal efficiency of the engine.

[75.67%; 8.68%; 11.479%]

26. The following data were obtained during a test on a single cylinder, double-acting, steam engine having 21 cm cylinder diameter, and 26 cm piston stroke:

Effective radius of the brake wheel, 38 cm; engine speed, 5 r.p.s.; net brake load, 1,334 newtons; m.e.p., 235 kPa; pressure of steam supplied, 8 bar; dryness fraction of steam supplied, 0-9; steam consumption, 180 kg per hour; condensate temperature, 60°C. Calculate: (a) the brake power, (b) the indicated power, (c) the mechanical efficiency, (d) the brake thermal efficiency, and (e) the indicated thermal efficiency.

[(a) 15-92 kW; (b) 21.14 kW; (c) 75-3%; (d) 13.79%; (e) 18.31%]

27. The following data were obtained during a test on a steam engine plant:

Indicated m.e.p., 2-5 bar; bore, 25 cm; stroke, 30 cm; r.p.m., 104; brake-torque, 932 N-m; pressure of steam supplied, 7 bar; steam supplied dry saturated; condenser vacuum, 610 mm of Hg; barometer reading, 760 mm of Hg; steam consumption, 2.7 kg per minute; 22.5 kg of coal with calorific value of 33,500 kJ/kg is supplied per hour. The steam engine is single-cylinder and double-acting.

Determine the mechanical efficiency and brake thermal efficiency of the steam engine. Also determine the overall efficiency of the steam engine plant (from coal to brake).

[79.53%; 8.96%; 4.85%]

28. The following data were obtained during a test of single-cylinder, double-acting steam engine having 21 cm cylinder diameter and 26 cm stroke:

Effective radius of brake wheel, 38 cm; Speed, 300 r.p.m.; Net brake load, 1,335 newtons; Mean effective pressure 2-4 bar; Pressure of steam supplied, 8 bar; Dryness fraction of steam supplied, 0.97; Steam consumption, 3.6 kg/minute; Condensate temperature, 60°C.

Calculate: (a) Brake power, (b) Indicated power, (c) Mechanical efficiency, (d) Brake thermal efficiency, (e) Indicated thermal efficiency, and (f) Steam consumption in kg per kW per hour based on brake power.

[(a) 15-937 kW; (b) 21.613 kW; (c) 73.74%; (d) 10.81%; (e) 14.66%; (f) 13.55 kg]

29. A double-acting, simple steam engine runs at 180 r.p.m. The cylinder diameter is 30 cm, the piston rod diameter is 5 cm, and the stroke length is 45 cm. Indicator diagrams taken on the engine show that the mean effective pressure is 4.5 bar for the in-stroke cycle and 4.7 bar for the out-stroke cycle. The engine is supplied with 1,000 kg of dry saturated steam per hour at 10 bar. The back pressure is 0.15 bar.

Calculate: (a) the indicated power, and (b) the indicated thermal efficiency of the engine.

[(a) 86-571 kW; (b) 12.21%]

30. Compare the Carnot and Rankine cycles using steam. Sketch p – v and T – ø diagrams for each cycle. Explain why Rankine cycle is employed rather than Carnot cycle as the standard of comparison for actual steam engine. Find the efficiency of Carnot cycle, using water and steam, between temperature limits corresponding to pressure of 14 bar and 1.4 bar. Compare the results with the efficiency of an engine following the Rankine cycle and working between the same pressure limits and using steam 90 per cent dry.

What influence has increasing wetness upon the efficiency of the Rankine cycle?

[Carnot η = 18.32%; Rankine η = 16.91%]
31. In what respect does the Rankine cycle differ from the Carnot cycle?

A prime-mover operating on the Rankine cycle is supplied with steam at pressure of 9 bar and dryness fraction 0.9, and exhausts at 0.7 bar. Find the work done in kJ per kg of steam and the efficiency of the Rankine cycle. Use the steam tables only. [385-14 kJ/kg; 17.56%]

32. Describe briefly the Rankine cycle with complete expansion and sketch the $p-v$ and $T-\Phi$ diagrams for the cycle. Calculate, using the steam tables only: (a) the ideal steam consumption in kg/hr. and (b) the thermal efficiency of the engine working on Rankine cycle and developing indicated power of 1,471 kW. The steam supplied is dry saturated at 24 bar and exhaust is at 0.15 bar. [(a) 6,870 kg/hr; (b) 29.91%]

33. A multiple expansion steam engine develops indicated power of 1,180 kW and uses 10,000 kg of steam per hour. The steam is supplied at 16 bar and 295°C and exhaust is at 647.5 mm of Hg (barometer 760 mm of Hg). Estimate: (a) the indicated thermal efficiency, (b) the efficiency of the Rankine cycle, and (c) the efficiency ratio or relative efficiency. Take $k_p$ of superheated steam as 2.3 kJ/kg K. [(a) 15.26%; (b) 28.56%; (c) 53.43%]

34. Explain briefly, with the help of $P-v$ and $T-\Phi$ diagrams, the essential differences between the Carnot and Rankine steam engine cycles.

An engine working on Rankine cycle receives dry saturated steam at 14 bar directly from the boiler and exhausts at 0.07 bar. Find the work done per kg of steam and the thermal (Rankine) efficiency. If the steam is throttled to 8.5 bar before entering the engine, find the reduction in work done per kg of steam and in thermal efficiency by means of steam tables and $H-\Phi$ chart. Take $k_p$ of superheated steam as 2.1 kJ/kg K. [781.2 kJ/kg; 29.74%; 65.8 kJ/kg, 24.8%]

35. A simple Rankine cycle steam power plant operates between the temperature limits of 260°C and 95°C. Steam is supplied to the turbine in a condition of dry saturated and the expansion in the turbine is isentropic. Draw the ideal Rankine cycle and the Carnot cycle using steam and capable of working between the same temperature limits on a $T-\Phi$ diagram. Estimate and compare the efficiencies of the two cycles while working between the given conditions.

Give reasons for difference in the values of the efficiencies of the two cycles. [Carnot $\eta$ = 30.9%; Rankine $\eta$ = 27%]

36. A steam engine working on the incomplete Rankine cycle is supplied with dry saturated steam at 10 bar. Pressure at release is 2.5 bar and exhaust takes place at 1.2 bar. Assuming isentropic condition with constant volume conditions between release and commencement of exhaust, determine using steam tables, (a) the work done per kg of steam, (b) the mean effective pressure, and (c) the efficiency of the cycle. Compare these values with those of Rankine cycle working between the same pressure limits.

[Incomplete Rankine : (a) 333.41 kJ/kg; (b) 5074 bar, (c) 14.26%; Rankine : (a) 363.88 kJ/kg; (b) 2895 bar, (c) 15.56%]

37. An engine receives steam at 18 bar with 50°C superheat. Expansion takes place at constant entropy to pressure of 2.8 bar. Exhaust is at 0.15 bar. Estimate: (a) the heat received per kg of steam, (b) the work done per kg of steam, (c) the heat rejected per kg of steam, and (d) the thermal efficiency of the cycle. Take $k_p$ of superheated steam as 2.1 kJ/kg K. [(a) 2,676.17 kJ/kg; (b) 511.17 kJ/kg; (c) 2,165 kJ/kg; (d) 19.1%]

38. In a modified Rankine cycle, steam is supplied at 14 bar with 38°C superheat. It expands isentropically to 2 bar and then released at constant volume to the exhaust pressure of 1 bar. Determine, using steam or $H-\Phi$ chart, (a) the thermal efficiency of the cycle, and (b) ideal steam consumption in kg per kW per hour. Take $k_p$ of superheated steam as 2.1 kJ/kg K. [(a) 17.85%; (b) 8.22 kg]

39. An engine is supplied with dry saturated steam at 11 bar. Expansion is carried out at constant entropy to a pressure of 2 bar and then released at constant volume to an exhaust pressure of 0.15 bar. Sketch the ideal pressure-volume and temperature-entropy diagrams for the cycle and indicate thereon the work done and the condition of steam at exhaust. Using the steam tables only, find the index of expansion, and the work done in kJ per kg of steam. Taking Rankine cycle as standard, find the percentage loss of work due to incomplete expansion. [1.137; 446.85 kJ/kg; 32.02%]

40. (a) Discuss the causes of loss of thermal efficiency in a steam engine.

(b) State the cause and effect of "initial condensation" in the cylinder of a steam engine.

(c) How do you calibrate an indicator diagram of a steam engine? What is a saturation curve?
41. Explain what do you understand by the term missing quantity as supplied to a steam engine? What are the causes of missing quantity and what measures are taken to reduce in its practice?

The total mass of steam in an engine cylinder is 0.037 kg. From the indicator card it is found that at a point during expansion stroke the pressure is 3.5 bar and volume occupied by steam is 0.014 m³. Calculate the dryness fraction of the steam and also the missing quantity in kg per stroke at this point.

\[ \text{Dryness fraction} = \frac{3.5}{0.014} = 0.025 \]  
\[ \text{Missing quantity} = 0.0103 \text{ kg} \]

42. Discuss in detail the different methods employed for reducing cylinder condensation in reciprocating steam engines.

In a double-acting steam engine, the measured mass of steam per minute was 245 kg, and speed was 110 r.p.m. The volume immediately after cut-off was 0.13 m³ and the pressure at this point was 12 bar. The volume immediately after compression had begun was 0.04 m³ and the pressure was 3.5 bar. Find the missing quantity in kg per minute and as a percentage of the actual cylinder feed at cut-off. The volumes given in the question are the total volumes at the points referred to.

\[ \text{Missing quantity} = 86.68 \text{ kg}; 35.38\% \]

43. Explain the term “missing quantity” and state its causes and methods adopted for reducing it.

A non-condensing, double-acting steam engine has a cylinder diameter of 25 cm and 50 cm stroke and the clearance volume is 9 per cent of the swept volume. The steam supplied is dry saturated and the engine runs at 100 r.p.m. At point A on the expansion curve immediately after cut-off and at 40 per cent of the outward stroke, the pressure taken from the indicator card is 5 bar. At a point B on the compression curve at 90 per cent of the return (exhaust) stroke the pressure is 1.4 bar.

Determine the indicated and actual steam consumptions per hour if the missing quantity at point A is 0.01 kg per stroke.

\[ \text{Indicated steam consumption} = 338.88 \text{ kg} \]  
\[ \text{Actual steam consumption} = 456.88 \text{ kg} \]

44. A double-acting, steam engine running at 75 r.p.m. has a piston diameter of 70 cm and 120 cm stroke and the clearance volume is 7 per cent of the swept volume. The pressures at cut-off and beginning of compression are 10 bar and 3.5 bar respectively.

Assuming that the expansion follows the law \( PV^{1.2} = C \) and the engine is supplied with 150 kg of steam per minute, estimate: (i) the pressure of steam at 0.5 of the expansion stroke, and (ii) the dryness fraction of the steam and the missing quantity per hour at 0.5 of the expansion stroke.

\[ \text{(i) Pressure} = 7.185 \text{ bar}; \text{(ii) Dryness fraction} = 0.803, \text{ Missing quantity} = 2.137 \text{ kg/hr} \]

45. Draw graphs showing how the steam consumption is kg/hour varies with indicated power in a simple steam engine governed (a) by throttling the entering steam, and (b) by varying the point of cut-off.

46. Sketch the shape of graph you would expect to obtain if steam consumption in kg/hour were plotted to a base of indicated power in kW during the trial of a throttle governed steam engine running at constant speed.

A throttle governed steam engine running at constant speed uses 350 kg of steam per hour when developing indicated power of 18.4 kW and 700 kg per hour when developing indicated power of 44 kW. Esimate the indicated thermal efficiency of the engine developing indicated power of 22 kW assuming that the steam supplied is dry saturated at 11 bar and exhaust is at 0.3 bar.

\[ \text{Indicated thermal efficiency} = 7.96\% \]

47. State what is meant by “Willan’s line”?

A steam engine governed by throttling and running at constant speed uses 600 kg of steam per hour when developing indicated power of 41 kW and 2,500 kg of steam per hour when developing indicated power of 214 kW. Derive the Willan’s law of the engine and find the steam consumption (i) in kg per hour, and (ii) in kg per kW-hour when developing indicated power of 148 kW.

\[ S = 150.28 + 10.98 \times (i) 1,775.32 \text{ kg/hr}; \text{(ii) 11,995 kg/kW-hr.} \]

48. State what is “Willan’s straight line law”?

A throttle governed steam engine running at a constant speed uses 14.3 kg of steam per kW per hour when developing indicated power of 7.4 kW and 10.9 kg per kW per hour when developing indicated power of 37 kW. Calculate the indicated thermal efficiency of the engine when developing indicated power of 22 kW. Dry saturated steam at 13 bar is supplied and temperature of condensate is 52°C.

\[ \text{Indicated thermal efficiency} = 12.2\% \]

49. Explain the terms: (i) Willan’s law, and (ii) Willan’s line.

A throttle governed steam engine running at a constant speed consumes 1,638 kg of steam per hour when developing indicated power of 206 kW and 2,725 kg of steam per hour when developing indicated power of 375 kW. Determine the efficiency ratio for the steam engine when developing indicated power of 295 kW. The steam is supplied at 16 bar and 300°C and the back pressure is 0.4 bar. Take, \( h_p \) of the superheated steam as 2.3 kJ/kg K.

\[ \text{Efficiency ratio} = 71.76\% \]