1.1 Introduction

Heat is defined as a form of energy in transition which flows under the driving influence of a temperature difference. Once the transfer of heat energy is completed, it is stored in one or more forms of energy of storage – potential, kinetic, or in general as internal energy. It is worth noting that the heat as energy in transition is never measured as such but is determined in terms of observed changes in other forms of energy and the relevant physical properties. Strictly speaking the heat transfer occurs only at boundaries of system while heat transfer anywhere else in a system is merely redistribution of internal energy within the system. There are three basic mechanisms of heat transfer — conduction, convection and radiation. In engineering application they may occur separately, or simultaneously.

1.1.1 Modes of Heat Transfer: Heat is transferred by conduction, convection and radiation.

Conduction is recognised as the transfer of heat within a substance from high temperature regions to low temperature regions. Conduction in solids other than metals is due to longitudinal oscillations in metals due to diffusion of free electrons and in gases due to elastic impacts of molecules. As kinetic energy of motion is proportional to the absolute temperature, it is logical to imagine conduction as occurring by collisions of faster with slower moving molecules. This idea seems to be quite correct. In the case of gases, molecular interaction is responsible for energy transfer; however, in metals an electron gas rather than the molecules is the primary medium of energy transfer.

Convection involves the gross motion of the fluid (liquid and gas) itself with the result that fresh fluid is continually available for energy transfer. The physical movement of fluid generally involves smaller eddies which help in distributing heat energy. The state and nature of fluid flow is of great importance in convective heat transfer. The fluid is set in bodily motion either due to

- difference in density due to heating, i.e. buoyancy forces, this is the case of free convection, or
- external force such as fans, blowers, pumps, etc. giving rise to forced convection.

Convection is not actually a separate process, as conduction to or from the fluid is really what constitutes the heat transfer, and the movement of the fluid carries heat transferred to another location. A household water heating system is a good example of this type of heat transfer.

Radiation is transfer of heat energy by temperature excited electromagnetic waves emitted by vibrating electrons in the molecules of a material at the surface of a body. The quantity of heat radiated depends on the absolute temperature of the body.

All bodies at temperatures above absolute zero emit electromagnetic waves of different wave lengths. Radiation differs from conduction and convection in this respect and is
distinguished by double transformation of energy-thermal energy is first converted into radiant energy by an emitter and then radiant energy into thermal energy by an absorber.

An interesting example of combined mode of heat transfer is a steam boiler. Here, heat is transferred from the flue gases to the outer surface of the water tubes through all the three modes of the transfer - conduction, convection and radiation. From the outer surface of a water tube to its inner surface, heat is transferred only by conduction through a layer of soot, metal wall and a layer of deposited scale. Finally, from the inner surface of the tube to the water, heat is transferred only by convection. Individual modes of heat transfer are, therefore, met with various combinations in the course of heat flow, and it is very difficult to separate them. In practical calculation, it is sometimes, desirable to consider such complex process as a whole.

![Fig. 1-1](image)

1.1.2 Irreversibility in Heat Transfer: It may be noted that transfer of heat is on account of temperature gradient existing between two bodies, which makes the process irreversible, i.e. flow of heat cannot be reversed of its own. Thus, heat transfer in the direction of temperature gradient is a natural and spontaneous process. As it happens in all natural processes, entropy increases in the system of bodies among which heat transfer takes place. Let there be two bodies A and B at temperatures $T_1$ and $T_2$. $T_1 > T_2$ as shown in fig. 1-1. If $dQ$ is the quantity of heat lost by A and that gained by B, entropy lost by A is $\frac{dQ}{T_1}$ and that gained by B is $\frac{dQ}{T_2}$.

Thus, in the system of two bodies due to heat transfer there is net increase of entropy $\frac{dQ}{T_2} - \frac{dQ}{T_1}$. The gain in entropy during heat transfer indicates fall in quality of heat energy. It is desirable to minimise irreversibility and also gain in entropy in any process in thermodynamics as it renders the process inefficient. This is accomplished by limiting temperature difference ($T_1 - T_2$) in the process of heat transfer. However, with the decrease of temperature gradient, the rate of heat flow decreases and eventually ceases when temperature equalization is attained. Further, with the system of bodies at different temperatures, on attainment of equalization heat transfer stops and the system attains uniform temperature. If the plot of increase in entropy of the system is plotted with time, it indicates decreasing rate of increase in entropy. On equalization, entropy of the system becomes maximum and no further increase in entropy of the system is feasible.

1.1.3 Fields of Applications: The importance of heat transfer analysis lies in a very wide range of applications connected with power plant engineering, chemical and process engineering, manufacturing and metallurgical industries, refrigeration and air conditioning
practices, cooling problems associated with electrical and electronic equipments, space technology, low temperature technology and many other applications. Condensers, evaporators, coolers, heat dissipating surfaces such as fins, prevention of heat losses through insulating materials, controlled release of heat energy from fossil and nuclear fuels, aerodynamic heating, combustion processes, thermally operated controls, etc. are some of the examples of heat transfer applications.

In the design of a plant which incorporates heat exchange with its surroundings, the size of the heat transfer equipments, selection of materials of construction connected with them, and the auxiliary equipment required for their utilization, are basic considerations being faced by a designer. The equipment should fulfil its required objectives and at the same time should be economical to purchase and operate. This requires thorough understanding of the basic mechanisms of heat transfer and analysis so as to be able to evaluate quantitatively heat transfer rates and other related quantities. Unfortunately, heat transfer analysis involves many variables and it is impossible to separate them and treat one at a time, e.g., the flow of heat through a condenser involves nine variables. This clearly requires detailed knowledge of the principles governing heat transfer.

Heat exchangers are very important parts for many thermal systems. Their first cost and the cost of their operation and maintenance are of great importance for economic plant installation. As stated earlier heat transfer is an irreversible process, hence it is desirable to reduce the irreversibility that always accompanies heat transfer by reducing temperature difference between the bodies that exchange heat. With the reduction of temperature difference, the rate of heat transfer decreases and in order to maintain the given rate of heat transfer, area of heat transfer is to be increased. Thus, cost of heat exchanger increases rapidly with the reduction in temperature difference employed for heat transfer. Hence, designers should make a decision regarding economic limit to which temperature difference can be reduced.

The popular examples of heat exchangers are:

(i) Surface condenser of a steam plant, (ii) Air preheater and economiser for a boiler, (iii) Intercooler for an air compressor, (iv) Heat exchanger for gas turbine plant, (v) Condenser and evaporator for a refrigeration unit, (vi) Heat exchanger for chemical plant, automobile radiator, etc.

In these classes of heat exchangers, the fluids are kept separate and heat transfer takes place through the intervening walls. This is often the only possible type as the fluids differ in their chemical composition and at least one is to be recirculated through the plant cycle so that they cannot be allowed to mix. Thus, in such case the rate of heat transfer between fluids is limited to the capacity of the separating wall to transfer heat. This capacity of heat transfer is the basis of the design for this type of heat exchanger.

1.2 Thermal Conduction

Heat transfer between a hot body and a cold body by conduction may take place through material substance such as metal wall, but conduction may take place also through fluids (either gases or liquids). In process of conduction, there is no physical movement of molecules. At the hot end of the material, random movements (activities) of the molecules is increased. As a result of this increased activity of the molecules, collisions with adjacent molecules along the material takes place imparting increased momentum to these adjacent molecules, resulting in increased temperature. This is transfer of heat by conduction. In gases, heat conduction occurs by molecules, and atomic interaction. In metals, the flow of energy is due mainly to the diffusion of free electrons. Here, crystal lattice vibrations are of secondary importance.
1.2.1 Basic Equation: The basic equation for rate of heat transfer by conduction for an elementary thickness \( dx \) of the plate is:

\[
H = -KA \frac{dT}{dx}
\]

where, \( H \) — rate of heat transferred, 
\( A \) — area of heat flow normal to the direction of heat flow, 
\( K \) — coefficient of thermal conductivity, and 
\( \frac{dT}{dx} \) — temperature gradient.

For positive value of \( x \), in the direction of heat flow, temperature decreases, making \( \frac{dT}{dx} \) negative. Hence, a negative sign is employed in eqn. (1.1) so that heat quantity becomes positive.

This basic equation although introduced by Biot in 1804, is usually attributed to Fourier because of his outstanding contribution to the field. This law was formulated from the study of experimental observations.

1.2.2 Thermal Conductivity: The thermal conductivity of the material, \( K \) is the quantity of heat passing between opposite faces of a unit cube in unit time when temperature difference is maintained across the faces. Thermal conductivity is a physical property of a substance and characteristics the ability of the substance to conduct heat.

The thermal conductivity, when expressed in S.I. units, is

\[
K = \frac{H/A}{dT/dx} \left[ \frac{\text{Watts}}{\text{m}^2\text{C}} \right] \quad \text{or} \quad \left[ \frac{\text{Joules}}{\text{s} \text{m}^2\text{C}} \right]
\]

This is the system of units which will be used throughout this chapter. In many cases (particularly for insulating materials) the temperature gradient is expressed in °C/cm, so that the unit of \( K \) becomes watts cm/m²°C. In MKS system of units, \( K \) may be expressed in kcal/hr m°C.

The thermal conductivity varies with temperature. Experiments show that for most materials, this dependence is linear, i.e.

\[
K = K_0 (1 + aT)
\]

where, \( a \) is a constant and \( K_0 \) is the value of thermal conductivity at 0°C. The constant \( "a" \) is positive for insulating materials and negative for metallic conductors. Magnesite, brass and aluminium are exceptions to this rule.

From values of the thermal conductivities of a few substances — solids, liquids and gases given in Table-1, it is seen that silver is the best conductor of heat. Mercury, though placed among liquids should be classified with metals on account of its comparatively high value. Liquids have low conductivity, but gases have extremely low values. Hydrogen appears to be the best conductor among gases; but helium has a slightly higher value.
### Table 1: Coefficient of Thermal Conductivity

<table>
<thead>
<tr>
<th>Substance</th>
<th>Thermal Conductivity $K$ (watts/m°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0.02 - 0.022</td>
</tr>
<tr>
<td>Alcohol</td>
<td>0.14 - 0.16</td>
</tr>
<tr>
<td>Aluminium</td>
<td>170 - 198</td>
</tr>
<tr>
<td>Asbestos</td>
<td>0.2 - 0.22</td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.6 - 0.65</td>
</tr>
<tr>
<td>Brass (70% Cu. 30% Zn)</td>
<td>90 - 120</td>
</tr>
<tr>
<td>Brick</td>
<td>155 - 165</td>
</tr>
<tr>
<td>Bonded silicon carbide</td>
<td>0.045 - 0.075</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.01 - 0.012</td>
</tr>
<tr>
<td>Chrome brick</td>
<td>0.9 - 1.9</td>
</tr>
<tr>
<td>Corrugated asbestos</td>
<td>0.075 - 0.075</td>
</tr>
<tr>
<td>Concrete dry</td>
<td>0.9 - 1.0</td>
</tr>
<tr>
<td>Copper</td>
<td>300 - 320</td>
</tr>
<tr>
<td>Chromium brick</td>
<td>20 - 24</td>
</tr>
<tr>
<td>Chromium-nickel steel</td>
<td>13.5 - 19.5</td>
</tr>
<tr>
<td>Ebonite</td>
<td>0.15 - 0.18</td>
</tr>
<tr>
<td>Glass wool</td>
<td>0.025 - 0.045</td>
</tr>
<tr>
<td>Glass</td>
<td>0.75 - 0.90</td>
</tr>
<tr>
<td>Hair, felt</td>
<td>0.3 - 0.45</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0.1 - 0.15</td>
</tr>
<tr>
<td>Iron</td>
<td>30 - 60</td>
</tr>
<tr>
<td>Lime stone</td>
<td>0.6 - 0.9</td>
</tr>
<tr>
<td>Lead</td>
<td>28.5 - 31.5</td>
</tr>
<tr>
<td>Magnesite brick</td>
<td>3 - 5.25</td>
</tr>
<tr>
<td>Mercury</td>
<td>7 - 8</td>
</tr>
<tr>
<td>Mica</td>
<td>0.6 - 0.65</td>
</tr>
<tr>
<td>Monel</td>
<td>27 - 30</td>
</tr>
<tr>
<td>Nickel</td>
<td>45 - 67.5</td>
</tr>
<tr>
<td>Oxygen</td>
<td>0.02 - 0.025</td>
</tr>
<tr>
<td>Platinum</td>
<td>60 - 75</td>
</tr>
<tr>
<td>Plaster</td>
<td>0.75 - 0.90</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.15 - 0.30</td>
</tr>
<tr>
<td>Silver</td>
<td>320 - 365</td>
</tr>
<tr>
<td>Silica brick</td>
<td>0.09 - 1.9</td>
</tr>
<tr>
<td>Water</td>
<td>0.4 - 0.5</td>
</tr>
<tr>
<td>Wrought iron</td>
<td>30 - 52.5</td>
</tr>
<tr>
<td>Zinc</td>
<td>82.5 - 97.5</td>
</tr>
</tbody>
</table>

#### 1.2.3 Conduction through a Plane Wall

Consider steady conduction through homogeneous wall of material of constant cross-sectional area and uniform thickness (fig. 1-2). The value of conductivity $K$ of the material is assumed constant. Constant temperatures $T_1$ and $T_2$ are maintained on the boundary surfaces of the wall. Temperature varies in the direction of $x$ axis perpendicular to the wall.

For steady conduction (refer eqn. 1.1),

$$H = -KA\frac{dT}{dx} = \text{Const.}$$

$$\therefore \frac{dT}{dx} = -\frac{H}{KA} = C$$

$$\therefore T = Cx + D$$

If $S$ is the thickness of the wall and measuring the distance from left hand face of
the wall,

at \( x = 0 \), \( T = T_1 \) and at \( x = S \), \( T = T_2 \);

Inserting these boundary values, the constants \( C \) and \( D \) are evaluated as

\[
D = T_1 \quad \text{and} \quad C = - \frac{(T_1 - T_2)}{S}
\]

\[
\therefore \quad T = T_1 - \frac{(T_1 - T_2)}{S} \times x
\]

... (1.3)

This indicates that if the conductivity of the material is constant, temperature distribution
is linear over the wall thickness as shown in fig. 1-2.

Now, consider conduction through a slab of material of constant cross-section area
(fig. 1-3). Allowing for the variation in the value of \( K \) with temperature as per eqn. (1.2)
for a layer of thickness \( dx \) located at the distance \( x \) from the outer surface,

\[
H = -KA \frac{dT}{dx} = -K_0 (1 + aT)A \frac{dT}{dx} \quad \therefore \quad \frac{Hdx}{AK_0} = -(1 + aT) dT
\]

Integrating both the sides,

\[
\frac{HS}{AK_0} = \left[ (T_2 - T_1) + \frac{a}{2}(T_2^2 - T_1^2) \right] = (T_1 - T_2) + \frac{a}{2}(T_1^2 - T_2^2)
\]

\[
\therefore \quad H = \frac{A(T_1 - T_2)K_0}{S} \left[ 1 + \frac{a}{2}(T_1 + T_2) \right] = \frac{A(T_1 - T_2)}{S} \times K \text{mean} \quad ... (1.4)
\]

where, \( K \text{mean} = K_0 \left[ 1 + \frac{a}{2}(T_1 + T_2) \right] \)

Hence, it is customary to work in conduction of heat transfer with arithmetic mean
value of \( K \) corresponding to the boundary wall temperatures.

Another way of writing eqn. (1.4) is

\[
H = \frac{T_1 - T_2}{S} = \frac{T_1 - T_2}{R} \quad \text{where,} \quad R = \frac{S}{AK \text{mean}}
\]

\( R \) is termed the thermal resistance to heat flow. This is analogous to Ohm’s law in
electricity, viz.

\[
\text{Current} = \frac{\text{Potential difference}}{\text{resistance}}
\]

Thus, \( \frac{I}{R} = G = \frac{AK}{S} \), also known as the thermal conductance of the material. It
represents the amount of heat conducted for a unit temperature drop.

The equation of the curve showing temperature distribution in the wall may be
expressed as

\[
T = -\frac{1}{a} + \sqrt{\left(\frac{1}{a} + T_1\right)^2 - \frac{2Hx}{ak_0A}}
\]

... (1.5)

This equation reveals that temperature in the wall actually changes along a curved
Problem - 1: The interior of an oven is maintained at a temperature of 860°C by means of suitable control apparatus. The walls of the oven are 45 cm thick and are constructed from material whose thermal conductivity is 0.261 watts/m°C. Estimate the heat loss for each square metre of wall surface per hour. The outside wall temperature is 250°C. Estimate also the resistance to heat flow.

Substituting the given values and dimensions in eqn. (1.4),

\[
H = \frac{KA(T_1 - T_2)}{S} = \frac{0.261 \times 1 \times (860 - 250)}{45} = 354 \text{ watts/m}^2
\]

This indicates that 354 watts of heat will flow through each square metre of the wall area.

The resistance to heat flow, \( R = \frac{T_1 - T_2}{H} = \frac{860 - 250}{354} = 1.721 \text{ m}^2\text{C/watts} \)

Problem - 2: Calculate the heat loss per square metre of surface area for a furnace wall 23 cm thick. The inner and outer surfaces are at temperature of 310°C and 40°C respectively. The thermal conductivity varies with temperature and is given by \( K = 0.4 + 10^{-6} T^2 \), where \( T \) is in °C and \( K \) is in watts/m°C.

Here, \( H = -KA \frac{dT}{dx} \) from eqn. (1.1)

For one square metre area, \( Hdx = -KdT = -(0.4 + 10^{-6} T^2) \) dT

Integrating over the wall thickness,

\[
HS = - \left\{ 0.4 \left( T_2 - T_1 \right) + \frac{10^{-6}}{3} \left( T_2^3 - T_1^3 \right) \right\}
\]

\[
\therefore H \times \frac{23}{100} = \left\{ 0.4 \left( T_1 - T_2 \right) + \frac{10^{-6}}{3} \left( T_1^3 - T_2^3 \right) \right\}
\]

i.e., \( H = \frac{100}{23} \left\{ 0.4 \left( 310 - 40 \right) + \frac{10^{-6}}{3} \left( 310^3 - 40^3 \right) \right\} = 512.6 \text{ watts/m}^2 \)

1.2.4 Conduction through Composite Plane Wall: Walls of several heterogeneous layers are called composite. Such are the walls of dwelling houses in which the bricks are covered with a layer of plaster on either side. Thus, walls of furnaces, boilers and other heat devices usually consist of several layers, viz., a layer of refractory, a layer of conventional bricks and a layer of heat insulation. Let us derive the calculation formula of conduction for such a composite wall.

Consider the heat conduction through composite wall of the two dissimilar substances in intimate contact (no contact resistance) having thermal conductivities \( K_1 \) and \( K_2 \) and thicknesses \( S_1 \) and \( S_2 \) as shown in fig. 1-4. Let \( T' \) be the interface temperature.

Considering first wall, \( H = \frac{K_1 A}{S_1} \left( T_1 - T' \right) \) .. (i)

Similarly, as the same quantity of heat flows through the second wall,
\[
H = \frac{K_2A}{S_2} (T' - T_2)
\]  

...(ii)

From (i) and (ii), \( T_1 - T' = \frac{H S_1}{K_1A} \)  

...(iii)

and \( T' - T_2 = \frac{H S_2}{K_2A} \)  

...(iv)

Adding (iii) and (iv), \( T_1 - T_2 = \frac{H}{A} \left( \frac{S_1}{K_1} + \frac{S_2}{K_2} \right) \)

\[ \therefore H = \frac{(T_1 - T_2)}{\frac{S_1}{K_1} + \frac{S_2}{K_2}} \]  

...(1.6)

This relationship is used in determining the rate of heat flow through composite walls.

The analysis can be extended for a composite wall consisting of more than two walls, say \( n \) walls.

For such a composite wall,  
\[ H = \frac{(T_1 - T_2)}{\sum_{i=1}^{n} \frac{S_i}{A K_i}} \]  

...(1.7)

As each term of the denominator of eqn. (1.6), and eqn. (1.7), represents the thermal resistance of a respective layer, it follows that the total thermal resistance of a composite wall is the sum of individual resistances.

Having obtained rate of heat flow (\( H \)) from eqn. (1.6), or (1.7), the interface temperatures can then be obtained from (i) or (ii), i.e. equation for an individual wall.

Sometimes, to simplify calculations, a composite layer is considered as homogeneous plane wall of a thickness \( \Delta \), and calculations are performed with the aid of the so-called equivalent thermal conductivity \( K_\theta \), whose value is determined by the following relation:

\[ H = \frac{T_1 - T_2}{\frac{1}{A} \sum \frac{S}{K}} = \frac{A K_\theta}{\Delta} (T_1 - T_2) \]  

...(1.8)

Thus,  
\[ K_\theta = \frac{\Delta}{\sum \frac{S}{K}} \text{ [watts/m°C]} \]

where,  
\( \Delta = \sum S = S_1 + S_2 + S_3 + \ldots + S_n \)

For a \( n \)-layer composite wall, we can write the following formula for equivalent conductivity:

\[ K_\theta = \frac{\sum_{i=1}^{n} S}{\sum_{i=1}^{n} \frac{S}{K}} \]  

...(1.9)

Equivalent thermal conductivity depends only on the thermal resistance and thickness of individual layers.

**Problem — 3**: A furnace wall is made up of a steel plate 1 cm thick, lined on the inner surface with silica bricks 15 cm thick, and on the outer surface with magnesite
bricks 15 cm thick. The temperature on the inner edge of the wall is 700°C and on the outer edge is 150°C. Find the quantity of heat lost in Joules per second per sq. metre and the temperature at interface of steel and magnesite bricks. Take the value of thermal conductivities for steel, silica brick and magnesite brick as 16-86, 1-74 and 5-23 J/m s °C respectively.

It is required that heat flow be maintained at 3,500 J/s m² by means of an air-gap between steel and magnesite bricks. Estimate the width of this air-gap, if K for air is 0-033 J/m s °C.

Using eqn. (1.7), 
\[ H = \frac{A(T_1 - T_2)}{\Sigma \frac{S}{K}} \]
where, \[ \Sigma \frac{S}{K} = \frac{1/100}{15-86} + \frac{15/100}{1-74} + \frac{15/100}{5-23} \]

\[ = 0-00059 + 0-0862 + 0-0286 = 0-11548 \frac{m^2s°C}{J} \]

∴ Heat flow \[ = \frac{1 \times (700 - 150)}{0.11548} = 4,762.72 \text{ J/m}^2\text{s} \]

Temperature drop in magnesite bricks \[ = 4,762.72 \times \left\{ \frac{S}{K} \right\}_{\text{MAG}} \]

\[ = 4,762.72 \times 0.0286 = 136.21 °C \]

∴ Interface temperature between steel and magnesite bricks \[ = 150 + 136.21 = 286.21 °C \]

In order to maintain the heat loss at 3,500 J/m²s, the value of \[ \Sigma \frac{S}{K} \] should be increased to \[ \frac{S}{K} = \frac{(T_1 - T_2)}{H} = \frac{(700 - 150)}{3,500} = 0-15714 \frac{m^2s°C}{J} \]

∴ \[ \frac{S}{K} \] for the air gap \[ = 0-15714 - 0-11548 = 0-04166 \frac{m^2s°C}{J} \]

∴ Air gap thickness \[ = 0-04166 \times 0-033 \times 1,000 = 1-3748 \text{ mm.} \]

### 1.2.5 Radial Flow through a Thick Cylinder Wall:

Numerous cases arise in practice, of heat flow from one fluid to another through the walls of a tube the temperature along the tube, being regarded as constant.

Consider a cylindrical wall (tube) \( l \) m long with inner radius \( r_1 \) m and outer radius \( r_2 \) m. The thermal conductivity of the material \( K \) is constant. The inner and outer faces are held at constant temperature \( T_1 \) and \( T_2 \) under the condition that \( T_1 > T_2 \) (fig. 1 - 5). Temperature varies only radially in \( r \) direction. Hence, the temperature field is one-directional. Take within the wall, an annular layer of thickness \( dr \) at a radius \( r \), bound by two cylindrical isothermal surfaces. The quantity of heat flowing through this layer per hour is,

\[ H = - K 2 \pi r l \times \frac{dT}{dr} \text{ watts} \]

Separating the variables and integrating between the radii \( r_1 \) and \( r_2 \),

![Fig. 1-5](image-url)
\[ H \int_{r_1}^{r_2} \frac{dr}{r} = -K2\pi I \int_{T_1}^{T_2} dT \]

\[ \therefore H \log_e \left( \frac{r_2}{r_1} \right) = 2\pi K I (T_2 - T_1) \]

i.e. \[ H = \frac{2\pi K I (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} = \frac{T_1 - T_2}{\log_e \left( \frac{r_2}{r_1} \right)} \]

Equating these two values of \( H \) we have, \( r_m = \frac{r_2 - r_1}{\log_e \left( \frac{r_2}{r_1} \right)} \) ... (1.11)

For composite cylindrical walls or pipes with lagging, equation (1.10) can be written as

\[ H = \frac{2\pi K I (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \]

\[ \Rightarrow \quad H = \frac{2\pi K I (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \]

If \( A_1 \) and \( A_2 \) are the inner and outer surface areas respectively of the cylinder, the appropriate mean area \( A_m \) is evidently,

\[ A_m = \frac{A_2 - A_1}{\log_e (A_2/A_1)} \]

This is called the logarithmic-mean area in terms of which the heat flow rate through a cylindrical wall is,

\[ H = KA_m \frac{T_1 - T_2}{r_2 - r_1} \]

When the value of \( A_2/A_1 \) does not exceed 2, the arithmetic mean area \( \frac{A_1 + A_2}{2} \) is accurate to within 4 per cent of the logarithmic-mean area. Thus, the use of arithmetic mean area in place of logarithmic-mean area in such cases, gives sufficiently accurate value of heat flow rate in engineering practice.

The temperature \( T \) of the cylindrical shell at the radius \( r \) can be shown to be given by
HEAT TRANSFER

\[ T = \frac{1}{\log_e (r_2/r_1)} \left[ (T_1 \log_e r_2 + T_2 \log_e r_1) - (T_1 - T_2) \log_e \frac{r_2}{r_1} \right] \quad \ldots (1.15) \]

If \( K \) is assumed to vary linearly as \( K = K_0 (1 + aT) \), the heat flow rate works out to be,

\[ H = \frac{2 \pi K_0 l}{\log_e (r_2/r_1)} \left[ 1 + \frac{a}{2} (T_1 + T_2) \right] (T_1 - T_2) \]

\[ = \frac{T_1 - T_2}{\log_e (r_2/r_1)} \frac{2 \pi K \text{mean}}{l} \quad \ldots (1.16) \]

1.2.6 Radial Heat Flow through a Thick Sphere : Using the same notations as for the cylinder, heat flow rate for an elementary sphere of radius \( r \) and thickness \( dr \) is,

\[ H = - K 4 \pi r^2 \frac{dT}{dr}, \quad \therefore H \int r^2 \frac{dr}{r^2} = - 4 \pi K \int r^2 dT \]

\[ = - H \left( \frac{1}{r_2} - \frac{1}{r_1} \right) = 4 \pi K (T_2 - T_1) \]

\[ \therefore H = \frac{4 \pi K (T_1 - T_2)}{\left( \frac{1}{r_1} - \frac{1}{r_2} \right)} = \frac{4 \pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1} = \frac{K \sqrt{A_1 A_2}}{(r_2 - r_1)} (T_1 - T_2) \quad \ldots (1.17) \]

The temperature \( T \) of spherical surface of radius \( r \) is given by

\[ T = \frac{1}{(r_2 - r_1)} \left\{ r_2 T_2 - r_1 T_1 + \frac{(T_1 - T_2) r_1 r_2}{r} \right\} \quad \ldots (1.18) \]

Problem – 4 : A steam pipe 10 cm outer diameter, is covered with two layers of insulating material, each 2.5 cm thick, the thermal conductivity of one being three times that of the other.

Working from first principles, show that the effective conductivity of the two layers is 15.7% less when the better insulating material is on the inside than when it is on the outside. Assume same overall temperature difference in both the cases.

Consider a thick cylinder to be made up of a number of elementary concentric cylinders of thickness \( dr \). If \( l \) is length of cylinder and \( K \), the thermal conductivity of material, rate of heat transmitted through elementary layer at radius \( r \) is,

\[ H = - K (2 \pi r l) \frac{dT}{dr} \]

Integrating between limits \( r_1 \) and \( r_2 \)

\[ H \int l \frac{dr}{r} = - 2 \pi K l \int \frac{dT}{T_1} \]

\[ \therefore H = \frac{2 \pi K l (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \]
For composite cylinder, \( H = \frac{2\pi l (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \)

\[ \Sigma = \frac{1}{K} \]

Let conductivity of the better insulating material be \( K \) and that of the other is \( 3K \).

With better insulating material on the inside,

\[ H_1 = \frac{2\pi l (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \log_e \left( \frac{r_3}{r_2} \right) \]

With better insulating material on the outside

\[ H_2 = \frac{2\pi l (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \log_e \left( \frac{r_3}{r_2} \right) \]

\[ \frac{H_1}{H_2} = \frac{1}{3} \log_e \left( \frac{3}{2} \right) + \log_e \left( \frac{4}{3} \right) \]

\[ = 0.843 = 1 - 0.157 \]

:. By putting better insulating material inside, the heat conducted is reduced by 15.7% than that conducted by placing it on outside.

**Problem — 5:** A sphere of radius 40 cm is lagged to a radius of 50 cm, the inner and outer surface temperatures of lagging being 230°C and 65°C respectively.

Find the rate of heat leakage, if coefficient of thermal conductivity is 5.5 watts per \( m^2 \) per °C difference per cm thickness.

Using eqn. (1-17),

\[ H = \frac{4\pi K (T_1 - T_2)}{r_2 - r_1} \]

\[ = \frac{4\pi \times 5.5 (230 - 65)}{100} \times \frac{40}{100} \times \frac{50}{100} \times \frac{100}{50 - 40} = 228 \text{ watts} \]

**Problem — 6:** A pipe having 16.5 cm external diameter and carrying brine, is lagged with 3.5 cm thickness of lagging for which coefficient of conductivity is 0.04 J/m s°C. Outer surface temperature of the lagging is 35°C and brine temperature at a section inside the pipe is −21°C. Find the rise in temperature of the brine per metre length at this section, if the brine flow rate is 0.32 kg/sec. Specific heat of brine is 3.6 kJ/kg°C.

Now, \( d_2 = 16.5 + (2 \times 3.5) = 23.5 \text{ cm} \), and \( d_1 = 16.5 \text{ cm} \).

Neglecting the thermal resistance of the pipe and using eqn. (1-10),

\[ H = \frac{2\pi K l (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \]
\[ H = \frac{2\pi K1(T_1 - T_2)}{\log_e \left( \frac{d_2}{d_1} \right)} \]
\[ = \frac{2 \times \pi \times 0.04 \times 1 \times ( - 21 - 35 )}{\log_e \left( \frac{23.5}{16.5} \right)} \]
\[ = -39.79 \text{ J/s per metre length of pipe.} \]

Negative sign indicates that heat flows from outer radius \( r_2 \) to inner radius \( r_1 \) of the lagging.

\[ \therefore \text{Heat gained by the brine is} \ 39.79 \text{ J/S per metre.} \]

Thus, rise of temperature of brine = \[ \frac{39.79}{0.32 \times 3.6 \times 1,000} = 0.0345^\circ \text{C} \]

### 1.3 Thermal Radiation

When heat exchange between any two bodies, situated at a distance apart, takes place even when no material substance fills the space between them, the process is called heat transfer by radiation. This transfer of heat takes place with the help of electromagnetic waves. These waves can also pass through material substances. The perfect transmitter of radiant heat is a vacuum; the ability of matter, whether solid or fluid, to transmit the wave motion tends to increase with the transparency of the material body. The intensity of the radiation emitted by the matter, and its distribution among the different wave lengths (\( \lambda \)) depends on the nature and temperature of the matter.

Radiation from a heated body may extend over a wide range of wave lengths, from short ultraviolet, through the visible range and into the long infrared heat rays. The properties of these rays are different. We are interested most in rays which are absorbed by substances and energy of which turns into heat energy in the course of absorption. Visible light rays and infrared rays (0.8 to 100 microns) possess such properties in the greatest measure. These rays are known as heat rays and the process of their propagation as thermal radiation or radiation.

Since, the nature of heat and visible light rays is one and the same, their physical properties are essentially similar too. The only difference is in their wave lengths. The wave length of visible rays ranges from 0.4 to 0.8 microns and that of heat rays ranges from 0.8 upto 100 microns, which includes ultraviolet rays and infrared rays as well.

The total radiant energy emitted per unit time per unit surface area from the body is defined as the total emissive power and is denoted by \( E \). It is the quantity which is directly proportional to the fourth power of the absolute temperature. In view of the complexity of the phenomenon of radiation, the fact that the overall effect can be simply represented as a fourth power function of the absolute temperature, stands as one of the wonders of nature.
As stated above, the amount of energy \( E \lambda \) emitted by body generally varies with wave length or frequency in the manner shown in fig. 1 - 6. This variation can be described by defining a *monochromatic emissive power* \( E \lambda \), such that the amount of energy emitted per unit time per unit area in the spectral range \( \lambda \) to \( \lambda + d\lambda \) is given by \( E \lambda \cdot d\lambda \). Thus, the total rate of energy emission at any temperature is given by area under the curve for that temperature, i.e.,

\[
E = \int_{0}^{\infty} E\lambda \cdot d\lambda
\]  

... (1.19)

1.3.1 Reflection, Absorption and Transmission of Radiation: Radiation is the property of all substances, and each continuously emits energy. In general, radiation falling on a body is partially reflected (scattered) at the surface. Waves that are not reflected, penetrate into the material and are progressively absorbed thereby producing heat. The remainder are transmitted through the body. This is illustrated in fig. 1-7.

For a given wave length \( \lambda \), the portion reflected at the surface depends on the material, the surface finish, and the angle of incidence, while for a given body and surface finish, the portion reflected also differs for different wave lengths. Also out of the radiation penetrating the surface, the amount absorbed depends on the material and the wave length of radiation.

For example, when thermal radiation from an electric fire (comprising of red light waves, \( \lambda_1 \) and infrared heat waves, \( \lambda_2 \)) fall on a slab of glass, the light waves are transmitted virtually undiminished while the heat waves are largely absorbed. Thus, reflection and absorption are both selective as regards wave lengths.

Now, in general the reflected energy impinges upon other (surrounding) bodies and forms the part of incident radiation on these bodies. This also happens to the fraction of the incident radiation transmitted through the body. Thus, after a series of absorptions, radiant energy is fully distributed among the surroundings bodies. Hence, each body not only emits radiant energy continuously but also absorbs it continuously.

The process of *radiant heat exchange* is the result of these phenomena linked with the double reciprocal transformation of energy (thermal-radiant-thermal). The amount of heat emitted or absorbed is determined by the difference between thermal energy radiated and absorbed by the substance. This difference always exists if the bodies participating in the inter-change of radiant energy are at different temperatures.

At equal temperature, entire system is in the so-called dynamic or *mobile thermal equilibrium*. In this case, all the bodies in the system also emit and also absorb, but the amount of energy absorbed is equal to that emitted for each body.

Thus, when radiant energy falls on a body, part may be absorbed, part reflected, and the remainder transmitted through the body. In other words,

\[
\alpha + \rho + \gamma = 1
\]

where, \( \alpha \) = *absorptivity* or the fraction of the incident radiant energy absorbed,
\[ \rho = \text{reflectivity} \text{ or the fraction of the incident radiant energy reflected, and} \]
\[ \gamma = \text{transmissivity} \text{ or the fraction of the incident radiant energy transmitted through the body.} \]

Above fractions are dimensionless and vary from 0 to 1. If \( \alpha = 1 \), then \( \rho = 0 \) and \( \gamma = 0 \), which means that incident energy is entirely absorbed by the body. Such a body is defined as a black body. In practice, most solid bodies are such strong absorbers of the thermal radiation penetrating the surface, that none of the original radiation is transmitted. This is so because the absorption of radiation by liquids and solids occurs in a very narrow region near the surface. For good conductors of electricity, this is of the order of 1 micron (10^{-6} m). In electrical insulators, this thickness may be as much as thousand microns. However, this is still such a short distance that, in all cases, practically \( \gamma = 0 \). Such bodies are called opaque bodies.

Thus, for opaque bodies
\[ \alpha + \rho = 1 \]

If \( \rho = 1 \), \( \alpha = 0 \) and \( \gamma = 0 \); which means that all the incident radiant energy is reflected, e.g., in case of a mirror, and the body is called white body.

If \( \gamma = 1 \), \( \alpha = 0 \) and \( \rho = 0 \); which means that the entire incident radiant energy passes through the body, e.g., in case of glass. Such bodies are called transparent or diathermanous.

There are no absolutely black (\( \alpha = 1 \)), white (\( \rho = 1 \)) and transparent (\( \gamma = 1 \)) bodies in nature, and these conceptions are conditional when applied to real bodies.

1.3.2 Concept of Black Body: A black body is one which absorbs all radiation incident upon it, whatever be the wave length \( \lambda \). The implication is that a black body is perfectly non-reflecting and non-transmitting at all wave lengths. Actually no material with \( \alpha = 1 \) and \( \rho = \gamma = 0 \) exists. Even the blackest surfaces occurring in nature have reflectivity of about 1 per cent (\( \rho = 0.01 \)).

Thus, although a black body must be black in colour, this is not a sufficient condition. Kirchhoff, however, conceived the following possibility of making a practically perfect black body: If a hollow body is provided with only one very small opening and is held at uniform temperature, then any beam of radiation entering by the hole is partly absorbed, and partly reflected inside. The reflected radiation will not be able to find the outlet, but will fall again on the inner wall. There again, it will be partly absorbed and partly reflected and so on. By such a sequence of reflection, the entering radiation will be almost completely absorbed by the body, and none will be able to come out though the opening. Thus, a small opening provided in a hollow body of any shape acts as a black body as shown in fig. 1–8.

All substances emit radiation, the quality and quantity depending upon the absolute temperature and the properties of the material comprising the radiating body. It may be shown that, at a given temperature, good absorber of any particular wave length is also the good emitter at that wave length. Thus, since by definition a black body is a complete absorber of radiation at all wave lengths, it is also the best possible emitter of thermal radiation, i.e., it is a perfect radiator.

1.3.3 Planck's law of Emission of Radiant Heat: Planck formulated a law regarding emission of radiation from black bodies which states that, the total heat loss by radiation
from a black body depends only upon its absolute temperature. The distribution of radiation energy among the different wave lengths for various temperatures is as shown in fig. 1–9.

According to Planck's law, the magnitude of intensity of radiation depends on the temperature of the radiating surface and the wave length of radiation. The peak wavelength, \( \lambda_m \) (i.e., that most strongly emitted) becomes shorter with increase in temperature. This is indicated by the dotted curve passing through the peak of all curves.

Wien's law relates \( T \) and \( \lambda_m \) by the following expression:

\[
\lambda_m T = 2.898 \, \text{mm}^{-5} \, \text{K}
\]  \( \ldots (1.22) \)

Planck introduced the quantum theory, which enabled him to express the following relationship between \( E_\lambda \) and \( \lambda \) for a black body in such a manner that it satisfactorily fits the experimental results:

\[
E_\lambda = \frac{C_1 \lambda^{-5}}{e^{C_2 / \lambda T} - 1}
\]  \( \ldots (1.23) \)

where, \( \lambda = \) wave length, m,
\( T = \) absolute temperature of body, K,
\( C_1 = \) constant = \( 3.74 \times 10^{-16} \) W/m²,
\( C_2 = \) constant = \( 1.4388 \times 10^{-2} \), mK, and
\[ E_\lambda = \text{monochromatic emissive power, W/m}^3. \]

Planck's formula has been checked experimentally and theoretically and is accepted as an exact relationship for black body radiation.

When the right hand term of eqn. (1.23) is multiplied by \( d\lambda \) and integrated between limits \( \lambda = 0 \) and \( \lambda = \alpha \), it yields the Stefan-Boltzmann law.

1.3.4 Stefan-Boltzmann's Law of Total Radiation: In 1879, Stefan concluded from experimental data that the total radiation by the black body per unit area per unit time is proportional to the fourth power of the absolute temperature of the body. Few years later, Boltzmann derived the same law from a thermodynamical reasoning. It, likewise, can be derived from Planck's law of radiation of black body as suggested earlier. For a black body the total energy \( E \), radiated per unit surface area per unit time is represented by areas under curves of fig. 1–9. These areas vary as fourth power of absolute temperature, \( T^4 \). In honour of its discoverers, the law has been called the Stefan-Boltzmann law. The law is expressed as

\[ E = \sigma T^4 \quad \text{... (1.24)} \]

where, \( E = \) heat energy radiated per unit time per unit area in W/m\(^2\),

\[ T = \text{absolute temperature in K, and} \]

\[ \sigma = \text{Stefan-Boltzmann's constant} = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 \]

The value of the constant \( \sigma \) is arrived at by using Stefan's experimental results.

For convenience eqn. (1.24) may be written as

\[ E = 5.67 \left( \frac{T}{100} \right)^4 \text{ W/m}^2 \quad \text{... (1.25)} \]

Equations (1.24) and (1.25) hold good for black surfaces only.

1.3.5 Radiation from Non-Black Bodies: For non-black bodies (i.e., those which either reflect or transmit part of the incident radiation), the amount of radiation emitted, when heated, falls short of that given by the Stefan-Boltzmann law. For such bodies Kirchhoff's law states that

\[ \frac{\text{Emissive power of a body at a given temperature and at a given wave length}}{\text{Emissive power of a similar black body at the same temperature and at the same wave length}} = \varepsilon \]

where, \( \varepsilon \) = emissivity of non-black body for particular temperature and wave length.

The emissivity of a body has importance in radiation similar to that of conductivity in conduction. Generally, it is difficult, if not impossible, to estimate the emissivity of a surface with sufficient accuracy of a few percent because the emissivity depends to some extent on the behaviour of the surface, particularly as far as metallic surfaces are concerned. Since perfectly non-black surfaces are not attainable in engineering practice, the equation (1.25) established for black body radiation must be multiplied by the fraction \( \varepsilon \) known as emissivity, for non-black bodies.

Hence, for a non-black body, the radiation intensity curve for a given temperature falls below that for a black body, and in addition, is distorted as shown in fig. 1–10 due to variation of emissivity with wave length.

1.3.6 Concept of Gray Body: A gray body is defined as one in which radiation spectrum is continuous and radiation spectrum curve is similar to the corresponding curve of a black body at the same temperature, i.e. the ratio emissive power of a gray body to that of black body at a given temperature is constant for all wave lengths, that is, its intensity curves will be identical in form with those of fig. 1–9, but to a reduced vertical
scale. The ratios of ordinates of curve for black body and gray body gives emissivity. Experiments reveal that most engineering materials are gray bodies.

Thus, for a gray body, the total heat loss by radiation is given by

\[ E = \varepsilon \alpha T^4 \quad \text{... (1.26)} \]

1.3.7 Emissive power of Non-Black Bodies: For practical purposes eqn. (1.26) may be used for bodies not truly gray, but in such cases \( \varepsilon \) is the mean value of the emissivity which varies with the wave length at the temperature considered. The mean emissivity varies some what with temperature.

The emissivity \( \varepsilon \) of non-metallic bodies does not vary much at ordinary temperatures. Some approximate values of emissivity \( \varepsilon \) are given in table 1-2.

![Fig. 1-10](image)

Comparison of radiation intensity curves for black, grey and non-black bodies at some temperature.

### Table 1-2 Emissivity \( \varepsilon \) of non-metallic bodies

<table>
<thead>
<tr>
<th>Materials</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon, oil, iron oxide</td>
<td>0.8</td>
</tr>
<tr>
<td>Rubber, wood, paper</td>
<td>0.85 to 0.9</td>
</tr>
<tr>
<td>Roofing paper, enamel, porcelain brick (red, rough), marble, glass</td>
<td>0.91 to 0.94</td>
</tr>
<tr>
<td>Asbestos slate, lamp black, ice, water glass compound</td>
<td>0.95 to 0.99</td>
</tr>
</tbody>
</table>

Metallic smooth surfaces emit very little radiation at ordinary temperatures and the emissivity increases moderately at higher temperatures. A few values are given in table 1-3.

![Fig. 1-11](image)

Derivation of Kirchhoff's law.

### Table 1-3 Emissivity \( \varepsilon \) of polished metallic surfaces

<table>
<thead>
<tr>
<th>Metal</th>
<th>Emissivity ( \varepsilon ) at different temperatures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>500°C</td>
</tr>
<tr>
<td>Aluminium</td>
<td>0.04</td>
</tr>
<tr>
<td>Copper</td>
<td>0.04</td>
</tr>
<tr>
<td>Gold</td>
<td>0.02</td>
</tr>
<tr>
<td>Silver</td>
<td>0.01</td>
</tr>
<tr>
<td>Steel</td>
<td>0.07</td>
</tr>
</tbody>
</table>

1.3.8 Kirchhoff's Law: Gustav Kirchhoff in 1895, established the relationship between the emissivity and absorptivity of a body. Consider radiant interchange between two surfaces, one gray and the other absolutely black as shown in fig. 1-11. The surfaces, are arranged...
parallel to each other and as close that the radiation of one falls on the other. Let the temperature, emissive power and absorptivity of the gray and black surfaces be respectively \( T, E, \alpha \) and \( T_0, E_0, \alpha_0 \); \( \alpha_0 = 1 \) and \( T > T_0 \) is assumed here.

The energy emitted \( (E \text{ W/m}^2) \) by gray surface is fully absorbed by the black surface. In turn the black surface emits \( E_0 \text{ (W/m}^2) \). A portion \( \alpha E_0 \) of this energy is absorbed by the gray body and the remaining portion \( (1 - \alpha) E_0 \) is reflected by the gray body and fully absorbed by the black surface. Thus, radiant interchange between the two surface is,

\[
H = E - \alpha E_0 \text{ W/m}^2
\]

This is the amount of energy lost by gray surface and gained by black surface. Radiant interchange between two surfaces also takes place when \( T = T_0 \). In this case, the system is in dynamic thermal equilibrium and \( H = 0 \),

i.e. \( E = \alpha E_0 \) or \( \frac{E}{\alpha} = E_0 = \frac{E_0}{\alpha_0} \) as \( \alpha_0 = 1 \).

The above relationship can be extended, by considering different gray surfaces in turn, as follows:

\[
\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3} = \ldots = \frac{E_0}{\alpha_0} = \frac{E_0}{\alpha_0} = \sigma T_0^{-4} \]

Thus, for all bodies, the ratio of the emissive power to absorptivity is the same and is equal to the emissive power of a black body at the same temperature.

Now, according to eqn. (1.26), \( E = \varepsilon \alpha T^4 \) for gray bodies. Thus,

\[
E_1 = \varepsilon_1 \sigma T_0^4, E_2 = \sigma_2 \sigma T_0^4, \ldots \text{ for } T_1 = T_2 \ldots = T_0
\]

Substituting above values in eqn. (1.27) and simplifying, we get,

\[
\frac{\varepsilon_1}{\alpha_1} = \frac{\varepsilon_2}{\alpha_2} = \frac{\varepsilon_3}{\alpha_3} = \ldots = \frac{\varepsilon_0}{\alpha_0} = 1
\]

Thus, the emissive power of bodies increases along with their absorptivity. If the absorptivity \( \alpha \) of a body is low, its emissive power \( \varepsilon \) is low too. Therefore, good reflectors are poor emitters; for instance, the emissive power of an absolutely white body is zero. Further, good emitters are good absorbers. Kirchhoff’s law expressed by eqn. (1.27) is applicable to integrated or total radiation, but it may also be used for monochromatic radiation. Thus, for radiation at the same wave length and temperature the ratio of the emissive power to the absorptivity is the same for all bodies.

**Problem — 7:** Determine the emissive power of the surface of the Sun if it is known to be at a temperature of 5,700°C and if it is considered to be black body. Also estimate the wave length at which the Sun emits maximum radiation and calculate total radiant energy emitted by the Sun if its diameter is known to be 1.391 \( \times \) 10\(^8\) m.

Emissive power of Sun as a black body is given by eqn. (1.24), i.e.,

\[
E = \sigma T^4 \text{ where } \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4
\]

\[
E = 5.67 \times \left\{ \frac{5,700 + 273}{100} \right\}^4 = 72.16 \times 10^6 \text{ W/m}^3.
\]

Now, the wave length at which Sun emits maximum radiation is given by eqn. (1.22), i.e.,

\[
\lambda_m T = 2 \cdot 898 \text{ mm K} = 2,898 \mu\text{m K}.
\]
Finally, as surface area of Sun as a sphere of diameter $D$ is $A = \pi D^2$,

$$H = E\pi D^2 = 72.16 \times 10^6 \times \pi \times (1.391 \times 10^9)^2 = 4.39 \times 10^{26} \text{ Watts}$$

**Problem — 8:** Calculate heat loss in watts, from a 20 mm diameter opening in an electric furnace, if it is maintained at a temperature of 1,000°C. Assume that the opening acts as a black body and that ambient air is at 27°C.

Here, $T_1 = 1,000 + 273 = 1,273 \text{ K}$ and $T_2 = 27 + 273 = 300 \text{ K}$.

Now, heat exchange by radiation from a black body to a large surrounding is given by eqn. (1.25), i.e.,

$$H = \sigma A (T_1^4 - T_2^4)$$

where $\sigma = 5.67 \times 10^{-8} \text{ watts/m}^2 \text{ K}$

$$\therefore H = 5.67 \times \pi \left( \frac{20}{1000} \right)^2 \times \left\{ \left( \frac{1,273}{100} \right)^4 - \left( \frac{300}{100} \right)^4 \right\} = 46.63 \text{ watts}. $$

### 1.4 Thermal Convection

The exchange of heat from a wall to a fluid or from a fluid to a wall is very important in engineering heat transfer. Heat flow in fluid produces difference in temperature and hence in density of various sections of the fluid, and relative movement of the layers of fluid will result. When the fluid movement occurs in this manner, the transfer of heat is called free or natural convection. Often, when the fluid is pumped or blown through the passage of a heat exchanger, its movement is directly encouraged by energy supplied by the pump or fan and the heat flow associated with this is termed forced convection. In the heat exchangers, the heat transfer is between fluids through a separating metal wall and convective effects are of great important. It enters in various ways in the operation and performance of engines, boilers, turbines, etc. and it is probably the most widely used means of providing comfort for our body. The problem of calculating convective heat transfer is much more varied, complicated and difficult than for either conducted or radiant heat. It may be said that convection is not a basic method of heat transfer, as are conduction and radiation. However, convection affects to very great extent, the rate of heat transfer by conduction.

Heat lost by convection (natural or forced) is an extremely complex process and there is no really accurate general law for either process. It is possible to investigate experimentally and express scientifically the results, for any one geometrical configuration (using dimensional analysis), but each configuration has its own law.

**1.4.1 Basic Equation:** Heat transfer by convection occurs on walls of rooms, on the outside of warm and cold pipes, and between the surfaces and fluids of all types of heat exchangers.

The flow of heat through the fluid film that is assumed to adhere to the surface of any solid in contact (fig. 1-12), the following equation, which goes back to Newton, is in
HEAT TRANSFER

general use:

\[ H = h A(T_1 - T_2) \]  \hspace{1cm} \text{(1.29)}

where, \( H \) = rate of heat transfer, watts,
\( A \) = area of surface in contact, \( m^2 \),
\( T_1 - T_2 \) = temperature difference between
the surface and fluid, °C, and
\( h \) = film coefficient, W/m\(^2\)°C.

Eqn. (1.29) simply states that temperature difference, \( \theta = (T_1 - T_2) \) between the
surface and a fluid in contact with it, causes a steady heat flow, \( H \). The factor of
proportionality, \( h \), defined by this equation is called \textit{film coefficient of heat transfer or film conductance} or simply \textit{coefficient of heat transfer}. As a matter of fact, the film coefficient
\( h \) is function of many variables, such as shape, kind of dimensions of the surface,
direction, and velocity of the flow, temperature, density, viscosity, specific heat, thermal
conductivity and coefficient of thermal expansion of the fluid.

1.4.2 Free or Natural Convection: As already explained, the motion of the fluid
particles in natural or free convective heat transfer is due to the difference in densities
of the parts of fluid. Heat transferred from the surface of an insulated pipe to the still
air of a room is an example of heat transfer by natural convection. In this case, some
of the heat is transferred by radiation also.

The experimental data on natural convection for some cases have been correlated
by dimensional analysis and these correlations are useful in the solution of problems.
Approximate simplified equations which apply to a limited range of experimental data have
been developed for several cases. Some of these equations are listed here. It may be
noted that they are valid for only a limited range and should be used with considerable
reservation.

1.4.3 Forced Convection: As discussed earlier, where the bodily motion of the liquid
or gas imposed by external means is very great in comparison with that induced by the
buoyancy forces of natural convection, the heat flow between surface and the fluid is
found to be directly proportional to the temperature difference \( \theta \) defined above. Here, fluid
temperature varies through the fluid. In order to estimate \( \theta \), either the average temperature
along a diameter of the tube or the mixing-cup temperature is considered. The latter is
obtained by perfectly mixing the fluid passing the cross-sectional area of the tube in a
"cup" and measuring the average temperature. For turbulent flow the temperature of the
fluid increases or decreases sharply in a very thin layer next to the surface and is
practically constant throughout the path of fluid.

Forced convection is probably the most important method of heat transfer employed
in engineering practice, \textit{e.g.} heat exchangers, artificial draught in boilers, cooling of internal
combustion engines, etc.

1.4.4 Dimensional Analysis: Principle of dimensional analysis was first applied by
Nusselt to problems of heat transfer by convection. This has helped to very great extent
in understanding fully the process of convection (forced and free) and deriving more
general expressions of film coefficient which are applied to wide range of problems.

The principle of dimensionless grouping is used in different branches of engineering.
The basic principle upon which this is founded is dimensional homogeneity, \textit{i.e.} dissimilar
quantities cannot be added together to form a valid physical relation. Mathematically, this
means that, in any physical equation both sides of which can be written as power functions
or algebraic sum of power functions, the sum of the exponents of the basic units must
be same on left and right sides. Before applying this principle, it is essential to work what variables are involved. Correct selection of the variables is a matter of experience.

**Free convection**: In order to develop a mathematical expression for the film coefficient of free convection, it is necessary first to establish the factors which influence the conductivity and the thickness of the stagnant film that is assumed to adhere to the surface over which convection currents pass. According to the laws of fluid flow and of heat transfer by conduction it is expected that the film conductance may be influenced by the size and shape of the surface by thermal conductivity $K$, specific heat $k_p$, viscosity $\mu$, density $\rho$, and by the buoyancy forces on account of heating. The size and shape of the stream can be expressed in terms of significant dimensions of the wall say $d$; and buoyancy forces may be expressed in terms of coefficient of thermal expansion $\beta$, acceleration due to gravity $g$, and temperature difference between surface in the fluid $\theta$. Thus, the film coefficient of free convection may be stated in equation form as

$$h = f(d, \rho, \mu, k_p, K, \beta, g, \theta)$$

A mathematical expression for $h$ will now be developed by the method of dimensional analysis. According to the $\pi$ theorem, the equation may be expressed in the form,

$$h = A \cdot (d)\rho (\mu)\beta (K)g (K_p)\beta (\beta g \theta)$$

It may be noted that variables, $\beta$, $g$, and $\theta$ are grouped together to represent buoyancy force. All variables may be expressed in the fundamental terms of length $L$, mass $M$, time $T$ and temperature change, $\theta$. Each of the variables appearing in eqn. 1.30, when expressed dimensionally becomes

$$h = \frac{H}{TL^2 \theta} = \frac{FL}{TL^2 \theta} = \frac{M}{T^3 \theta} ; k_p = \frac{H}{M \theta} = \frac{FL}{M \theta} = \frac{L^2}{T^2 \theta} ;$$

$$d = L ; \quad \beta = \frac{1}{\theta} ;$$

$$\rho = \frac{M}{L^3} ; \quad g = \frac{L}{T^2} ;$$

$$\mu = \frac{FT}{L^2} = \frac{M}{LT} ; \quad \theta = \theta ;$$

$$K = \frac{H}{TL \theta} = \frac{FL}{TL \theta} = \frac{ML}{T^3 \theta}$$

Expressed by dimensions, equation (1.30) becomes,

$$MT^{-3} \theta^{-1} = A \cdot (L)^{\rho} (M)^{q} (M)^{r} (ML)^{s} (L^2)^{x} (\frac{L}{T^2})^{y}$$

From which, equating indices of basic dimensions on either sides, following simultaneous equations are obtained:

$$1 = q + r + s ; 0 = p - 3q - r + s + 2x + y ; -3 = -r -3s -2x -2y ;$$

$$-1 = -s -x$$

Since, there are seven quantities expressed in terms of four fundamental dimensions, there will be three dimensionless groups. Expressing $p, q, r$ and $s$ in terms of $x$ and $y$.

$$s = 1 - x ; r = 3 - 2x - 2y - 3 + 3x = x - 2y$$

$$q = 1 - r - s = 1 + 2y - x - 1 + x = 2y$$

$$p = 3q + r - s - y - 2x = 6y - 2y + x - 1 + x - y - 2x = 3y - 1$$
HEAT TRANSFER

\[ h = A \cdot (d)^{-1} + 3y \cdot (\rho)^{2y} \cdot (\mu)^{x} \cdot (K)^{1-x} \cdot (kp)^x \cdot (\beta g \theta)^y \]

\[ = A \left( \frac{K}{\rho} \right) \left( \frac{d^3 \rho^2 \beta g \theta}{\mu^2} \right)^y \left( \frac{\mu kp}{K} \right)^x \]

\[ \therefore \left( \frac{hd}{K} \right) = A \left( \frac{\mu kp}{K} \right)^x \left( \frac{d^3 \rho^2 \beta g \theta}{\mu^2} \right)^y \] ... (1.31a)

This equation is known as Nusselt's expression, and the dimensionless groups in bracket are known as follows:

\[ \frac{hd}{K} \] is termed Nusselt number, \( N_{nu} \) or boundary modulus.

\[ \frac{\mu kp}{K} \] is termed Prandtl number, \( N_{pr} \) or Prandtl modulus.

\[ \frac{d^3 \rho^2 \beta g \theta}{\mu^2} \] is termed Grashof number, \( N_{gr} \) or free convection modulus.

With these symbols eqn. (1.31a) may be written in the abbreviated form

\[ N_{nu} = A (N_{gr})^y \cdot (N_{pr})^x \] ... (1.31b)

Eqn. (1.31b) is verified experimentally for the free convection by many investigators.

Forced Convection: As stated earlier in case of forced convection the fluid flow is turbulent. The conductivity and film thickness of the stagnant film that is assumed to adhere to surface over which convection currents pass, are affected by the size and shape of the surface, the surface thermal conductivity \( K \), specific heat \( k_p \), viscosity \( \mu \), and density of the fluid \( p \) at the mean temperature of the film; and by velocity of the fluid stream \( V \). Thus, film coefficient may be stated in equation form as

\[ h = f (d, K, \rho, \mu, V, k_p) \]

or \[ h = A \cdot (d)^{p} \cdot (K)^{q} \cdot (\rho)^{r} \cdot (\mu)^{s} \cdot (V)^{t} \cdot (k_p)^{y} \] ... (1.32)

where \( A, p, q, r, s, x \) and \( y \) are constants different from those in eqn. (1.30).

Expressed by dimensions, equation becomes

\[ (MT^{-3} \theta^{-1}) = A(L)^{p} \cdot (MLT^{-3} \theta^{-1})^{q} \cdot (ML^{-3})^{r} \cdot (ML^{-1} T^{-1})^{s} \cdot (LT^{-1})^{t} \cdot (L^{2} T^{-2} \theta^{-1})^{y} \]

from which, the following simultaneous equations are obtained:

\[ 1 = q + r + s ; \]
\[ 0 = p + q - 3r - s + x + 2y \]
\[ -3 = -3q - s - x - 2y ; \]
\[ -1 = -q - y \]

Expressing \( p, q, r \) and \( s \) in terms of \( x \) and \( y \) we get,

\[ p = -q + 3r + s - x - 2y = -1 + y + 3x + y - x - x - 2y = x - 1 \]
\[ q = 1 - y \]
\[ r = 1 - q - s = 1 - 1 + y - y + x = x \]
\[ s = 3 - 3q - x - 2y = 3 - 3 + 3y - x - 2y = y - x \]

Substituting values of \( p, q, r \) and \( s \) in eqn. (1.30), we get,

\[ h = A (d)^{x} \cdot (K)^{1-x} \cdot (\rho)^{y} \cdot (\mu)^{x-y} \cdot (V)^{x} \cdot (k_p)^{y} \]

\[ = A \left( \frac{d \rho V}{\mu} \right)^{x} \left( \frac{\mu kp}{K} \right)^{y} \left( \frac{K}{\sigma} \right) \]
In the above equation, \( \frac{Vd\rho}{\mu} \) is known as Reynolds number or modulus, \( N_{re} \).

\[ N_{nu} = A (N_{re})^x (N_{pr})^y \] \hfill (1.33c)

Other two dimensionless numbers in use are Stanton number or modulus expressed as

\[ N_{st} = \frac{N_{nu}}{N_{re} \times N_{pr}} = \frac{h}{Vkp\rho} \]

and Peclet number expressed as

\[ N_{pe} = N_{re} \times N_{pr} = \frac{Vd\rho kp}{K} = \frac{Vd}{\alpha} \]

where, \( \alpha = \frac{K}{kp\rho} \) = thermal diffusivity of material as defined earlier.

For convective flow in general which may be partly by free convection and partly by forced convection, film coefficient may be expressed in dimensionless form as

\[ N_{nu} = f(N_{gr}, N_{re}, N_{pr}) \] \hfill (1.34)

A good general correlation of the data on heating or cooling of any fluid in turbulent motion (Reynold's number in excess of 2,300) through pipes is

\[ N_{nu} = 0.023 (N_{re})^{0.8} (N_{pr})^{y} \] \hfill (1.35)

where, for heating \( y = 0.4 \) and for cooling \( y = 0.3 \) may be used.

All physical quantities are taken as arithmetic mean of those at entrance and exit.

For different gases, it is found that the values of Prandtl number only varies between 0.65 and 0.85 and usually it is taken constant as 0.75.

1.4.5 Empirical Relations: Numerous empirical formulae are available for free and forced convections for different fluids in tubes and over plates. Attempt is made here to list a few out of them.

Free convective heat flow over vertical plates

The surface heat transfer coefficient is implicity given by the general equation,

\[ N_{nu} = k (N_{pr} \times N_{gr})^n \] \hfill (1.36)

The term fluid stands for gas, vapour, or liquid flow in which convection occurs. The properties of fluid are evaluated at the mean temperature of plate surface and the fluid.

In the laminar range, i.e., at \( 10^{4} < N_{pr} \times N_{gr} < 10^{9} \),

\[ k = 0.59 \text{ and } n = 0.25 \]

In the turbulent range, i.e., at \( 10^{9} < N_{pr} \times N_{gr} < 10^{12} \),

\[ k = 0.13 \text{ and } n = 0.33 \]

Free convective cross flow over horizontal pipes

For one plain horizontal pipe and daitomic gases, e.g., air,

\[ N_{nu} = 0.37 (N_{gr})^{1/4} \] \hfill (1.37)

Forced convective heat flow for turbulent flow parallel to plane plates

The basic equation reads,
Forced convective heat flow for air parallel to plain pipes

Jakob and Dow suggest that

\[ N_{nu} = 0.028 (N_{re})^{0.8} \]  

... (1.39)

where characteristic length in Nusselt's as well as Reynold's number is the length and not the diameter of the pipe in question.

Convective heat transfer with turbulent flow inside pipes, for Re < 10,000

For liquid flow basic equation reads

\[ N_{nu} = 0.024 (N_{re})^{0.8} (N_{pr})^{0.37} \]  

... (1.40)

For flow of gas or vapour the basic equation reads

\[ N_{nu} = 0.04 (N_{re} \times N_{pr})^{0.75} \]  

... (1.41)

For non-circular conduits hydraulic mean diameter is used for diameter of the pipe. When the pipe is not straight, Nusselt's number of \( h \) is multiplied by the factor

\[ f = \left( 1 + 1.77 \frac{d}{R} \right) \]  

... (1.42)

where, \( R \) = radius of curvature, m

and \( d \) = diameter of pipe, m.

Convective heat flow with laminar flow inside pipes, \( R_e \leq 2,200 \)

Basic equation reads

\[ N_{nu} = 3.3 + 0.4 \left( N_{re} \times N_{pr} \times \frac{1}{L} \right) \]  

where, \( L \) is the length of pipe, m.

The equation is valid for gases and vapours as well as liquids.

Convective heat transfer by turbulent cross flow of gases and vapours with plain pipes.

Experience has shown that cross flow results in higher surface transfer coefficient than does with parallel flow and it is, therefore, increasingly used in practice.

For turbulent flow, \( N_{re} > 2,300 \), the general equation reads,

\[ N_{nu} = k (N_{re})^{0.6} (N_{pr})^{0.33} \]  

where, \( k = 0.26 \) for 10 or more pipes in line, and 0.33 for 10 and more staggered pipes.

If the number of pipes is less than 10, the above equation must be multiplied by factor \( K \) according to the following table:

<table>
<thead>
<tr>
<th>Number of pipes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor, ( K )</td>
<td>0.64</td>
<td>0.76</td>
<td>0.83</td>
<td>0.87</td>
<td>0.92</td>
<td>0.94</td>
<td>0.96</td>
<td>0.97</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Convective heat transfer by turbulent cross flow of liquids

The basic equation reads as

\[ N_{nu} = 0.145 (N_{re})^{0.654} (N_{pr})^{0.31} \]  

where the pipes are in line,  

... (1.45)

and \[ N_{nu} = 0.175 (N_{re})^{0.68} (N_{pr})^{0.31} \]  

where the pipes are staggered.  

... (1.46)
Problem — 9: Estimate the heat loss by natural convection from a horizontal plate, 18 cm x 22.5 cm, at 150°C facing upwards to the still air at 27°C. The film coefficient for free convection is given by,

\[ h = 2.48 \times (\theta)^{\frac{1}{4}} \text{ watts/m}^2\text{°C} \]

where, \( \theta \) is the temperature drop in the air film in °C.

Here, \( h = 2.48 \times (150 - 27)^{\frac{1}{4}} = 8.261 \text{ watts/m}^2\text{°C} \)

Using eqn. (1.29), total heat loss by natural convection is,

\[ H = h \times A \times 0 \]

\[ = 8.261 \times \frac{18 \times 22.5}{10,000} \times (150 - 27) = 41.15 \text{ watts} \]

Problem — 10: An evaporator consisting of 42/40 mm pipes is immersed in ordinary water flowing normal to the pipes at the velocity 0.5 m/sec in the narrowest cross section. The surface temperature is 2°C and mean water temperature is 7°C. Calculate the surface heat transfer coefficient. The pipes are staggered.

Use the relation, \( N_{\text{nu}} = 0.175 (N_{\text{re}})^{0.68} (N_{\text{pr}})^{0.31} \)

Assume the properties of water at 4.5°C as \( K = 0.564 \text{ Joules/m s °C} \), \( v = 1.572 \times 10^{-6} \text{ m}^2/\text{sec.} \), \( k_p = 4.208 \text{ KJ/kg°C} \) and \( \rho = 1,000 \text{ kg/m}^3 \)

Now, \( N_{\text{re}} = \frac{\nu d}{v} = \frac{0.5 \times 0.042}{1.572 \times 10^{-6}} = 1.33 \times 10^4 \)

It may be observed that Reynolds's number is greater than 2,300 and hence the flow is turbulent.

Again \( N_{\text{pr}} = \frac{\mu k_p}{K} = \frac{\nu \rho k_p}{K} \)

\[ = \frac{1.572 \times 10^{-6} \times 1,000 \times 4.208 \times 1000}{0.564} = 11.7 \]

\( \therefore N_{\text{nu}} = 0.175 (1.33 \times 10^4)^{0.68} (11.7)^{0.31} = 211 \)

But, \( N_{\text{nu}} = \frac{hd}{K} \)

\( \therefore \) Film coefficient, \( h = \frac{N_{\text{nu}} \times K}{d} \)

\[ = \frac{211 \times 0.564}{0.042} = 2,833 \text{ Joules/m}^2\text{s°C} \]

2.5 Combined Effect of Basic Modes of Heat Transfer

So far only such cases of heat transfer have been considered where either conduction, convection or radiation occurred separately or the effect of the one was so prominent that the others can be disregarded. This is not generally true. In practice heat transfer is on account of combined effect of two or three modes of transfer. In practical calculations the division of such complex processes into elementary phenomenon is not always possible and desirable.

There are two main cases in which the combined effect of conduction, convection and radiation must be considered. The one deals with projection of the surface in air, e.g., fins on the cylinder of air compressors and aero engines. Here, heat is conducted from the
root to the free end and at the same time heat is given up to the surrounding by
c onvection and radiation. The radiation effect being small can be neglected or simply
considered as included in convection effect.

The second case occurs at the wall of buildings or in heat exchangers. Consider the
transfer of heat from the hot water to the cold water in a tubular water cooler. The
temperature of the hot water is reduced as it passes through the tubes owing to the
gradual dissipation of heat energy of the hot water particles to the cooler surface of the
wall which separates the hot and cold water by convection. The heat energy then flows
through the metal wall by conduction and finally into the cold water by convection.

The combination of conduction and convection modes of heat transfer together with
radiation also occurs in practice. The analysis of such complex problems as the heat
transfer within a furnace of generating unit, where conduction, convection, and radiation
are all involved, would indeed be very difficult. However, there are a few simple methods,
by which the influence of radiation together with conduction and convection may be
studied.

1.5.1 Heat transfer from a Rod Heated at One End: The combined effect of all
basic modes of heat transfer can be illustrated by giving an example of the transfer from
a rod of arbitrary constant cross-section projecting from a vessel at constant temperature.
Here, convection is usually regarded as the principal phenomenon. In this case the process
is characterised quantitatively by heat transfer coefficient \( h = h_c + h_r \), where, \( h_c \) accounts
for convection and \( h_r \) for radiation.

Heat loss from infinitely long bar: Consider an uniform cross-section bar of infinite
length heated at one end (at \( x = 0 \)) and losing most of the heat from the surface (fig
1-13) by free convection and helped by radiation to some extent.

Let \( \theta \) be the temperature of the bar at section A above that of the surrounding.
The temperature at \( x = 0 \) is maintained at \( \theta_1 \) degrees above that of the surrounding.
The area of cross-section and perimeter of the bar are \( a \) in sq. \( m \) and \( p \) in \( m \) respectively.
The heat flow at A in time $\delta t$ is,

$$H_1 = -K a \frac{d\theta}{dx} \delta t$$

At $A'$ i.e. $x + \delta x$, the temperature of the bar above surrounding is,

$$\theta + \frac{d\theta}{dx} \delta x$$

Heat conducted through section $A'$ is,

$$H_2 = -K a \frac{d\theta}{dx} (\theta + \frac{d\theta}{dx} \delta x) \delta t$$

:. The difference between heat entering at A and that leaving at $A'$ is

$$H_1 - H_2 = -K a \frac{d\theta}{dx} \delta t + K a \left( \frac{d\theta}{dx} + \frac{d^2 \theta}{dx^2} \delta x \right) \delta t = K a \frac{d^2 \theta}{dx^2} \delta x \delta t$$

At steady state condition, $H_1 - H_2$ is the free heat loss from the surface of the bar between section $A$ and section $A'$. If $h$ is free heat loss coefficient,

$$K a \frac{d^2 \theta}{dx^2} \delta x \delta t = h \rho \delta x \theta \delta t$$

:. $$\frac{d^2 \theta}{dx^2} = \frac{h \rho \theta}{K a} \text{ or } \frac{d^2 \theta}{dx^2} - \mu^2 \theta = 0 \text{ where } \mu = \frac{h \rho}{K a} \quad \text{... (1.47)}$$

The general solution of this second order differential equation is

$$\theta = A e^{\mu x} + B e^{-\mu x} \quad \text{... (1.48)}$$

where $A$ and $B$ are constants which can be determined from known boundary conditions.

For infinite bar, at $x = 0$, $\theta = \theta_1$ and at $x = \alpha$, $\theta = 0$,

From eqn. (1.48), $\theta_1 = A + B$ and $0 = A x + B x 0$

:. $A = 0$ and $B = \theta_1$

:. $$\theta = \theta_1 e^{-\mu x} = \theta_1 e^{-\frac{h \rho}{K a} \times x} \quad \text{... (1.49)}$$

Eqn. (1.49) can also be written as, $\text{log}_e \left( \frac{\theta_1}{\theta} \right) = \mu x \quad \text{... (1.50)}$

It may be noted that $\text{log}_e \left( \frac{\theta_1}{\theta} \right)$ and $x$ are linearly related. Thus, it is very easy to determine the value of $h$ experimentally. Readings of $\theta$ along the length of the bar are taken and plotted as $\text{log}_e \left( \frac{\theta_1}{\theta} \right)$ against $x$. A mean line is drawn through these points and gradient of this line is found, which gives the value of $\mu$ and hence value of $h$.

Heat loss from a finite length (L) of an infinitely long bar:

This heat loss can be estimated by two different methods:

(i) The rate of heat loss from the surface of the finite length (L) of infinite bar

:. $H = \left[ -K a (\frac{d\theta}{dx})_{x = 0} \right] - \left[ -K a (\frac{d\theta}{dx})_{x = L} \right] = -K a \left[ (\frac{d\theta}{dx})_{x = 0} - (\frac{d\theta}{dx})_{x = L} \right]$  

But from eqn. (1.49),
\[ H = -Ka \left[ -\mu \theta_1 + \mu \theta_1 e^{-\mu L} \right] \]
\[ = Ka \mu \theta_1 \left[ 1 - e^{-\mu L} \right] \quad \text{... (1.51a)} \]

Alternatively
(i) By integrating along the length \( L \) over the surface of the bar for free loss,
\[ H = \int_0^L h p \theta_1 \, dx = \int_0^L h p \theta_1 \, e^{-\mu x} \, dx = \frac{h p \theta_1}{\mu} (1 - e^{-\mu L}) \]

Using the relation, \( \mu^2 = \frac{h p}{Ka} \), it can be shown that
\[ H = Ka \mu \theta_1 \left( 1 - e^{-\mu L} \right) \text{ same as eqn. (1.51a)} \]
\[ = \sqrt{h p Ka} \theta_1 \left( 1 - e^{-\mu L} \right) \quad \text{... (1.51b)} \]

1.5.2 Formed Surface or fins: Idea of fins on cylinders of aero-engines, air compressors, electric motors, and radiators, etc. is to reduce the temperature of the surface to which they are attached, and the fin from the point of view of heat transfer corresponds to a bar of finite length (fig. 1-14).

Heat flow \[ H = Ka \theta_1 \mu \left[ \frac{h}{K \mu} + \tan h(\mu L) \right] \]
\[ \left\{ 1 + \frac{h}{K \mu} + \tan h(\mu L) \right\} \]

where, \( \mu = \sqrt{\frac{h \times 2(b + Z)}{KZb}} = \sqrt{\frac{h \times 2Z}{KZb}} = \frac{\sqrt{2h}}{Kb} \)

![Fig. 1-14 Rectangular fin.](image)

![Fig. 1-15 Variation of heat loss from fins with fin length.](image)
In any particular application, $K$, $a$, $\mu$, $\theta_1$, and $h$ are constants. Hence, $H$ is function of $L$ only i.e. $H = f(L)$

We are interested in the rate of increase of heat flow with increase in length ($L$) of the fins i.e. $\frac{dH}{dL}$

Fig. 1-15 shows the variation of $H$ with respect to $L$ for particular values of $\frac{2h}{Kb}$.

It may be noted that as $\frac{2h}{Kb}$ approaches unity, $\frac{dH}{dL}$ approaches 0 and the fins become ineffective.

Fins are effective in general, if $\frac{2h}{Kb}$ has values exceeding five.

It can also be proved (by taking actual value of $h$ for air and water) that in general, the method of cooling, using fins, is not effective with liquid cooling as it is with air and gas cooling.

**Problem — 11**: A 60 cm long metal bar of rectangular cross-section 20 cm $\times$ 1.25 cm has one of its end at uniform temperature of 425°C above that of the surrounding air. If temperature gradient at the other end of the bar is zero, find the heat loss per second from the bar.

Take coefficient of emissivity from the surface of the bar as 6.5 J/m² s °C and coefficient of thermal conductivity for the material of the bar as 40 J/m s °C.

Using eqn. (1.51a),

$H = Ka \mu \theta_1 \tan (\mu L) = Ka \mu \theta_1 \left[ \frac{e^{\mu L} - e^{-\mu L}}{e^{\mu L} + e^{-\mu L}} \right]$,

where, $\mu^2 = \frac{ph}{Ka} = \frac{2(0.2 + 0.0125) \times 6.5}{40 \times 0.2 \times 0.0125} = 27.6$

$\therefore \mu = 5.25$

Thus, $e^{\mu L} = e^{5.25 \times 0.6} = e^{3.15} = 23.4$,

giving $\left[ \frac{e^{\mu L} - e^{-\mu L}}{e^{\mu L} + e^{-\mu L}} \right] = \frac{23.4 - 1}{23.4 + 1} = 0.995$

Hence, $H = 40 \times (0.2 \times 0.0125) \times 5.25 \times 425 \times 0.995 = 222$ J/s

**Problem — 12**: Two long pieces of copper wire, 1.5 mm in diameter, are to be soldered together at the ends. If the melting point of the solder is 235°C, estimate the minimum heat input required in watts. Take environment temperature as 25°C and effective heat transfer coefficient over the wire surface as 15 watts/m² s °C. Thermal conductivity of the copper wire is 390 watts/m°C.

Here, $h = 15$ watts/m² s °C, $K = 390$ watts/m°C,

$\rho = \pi d = \pi \times \frac{1.5}{1,000} = 0.0047$ m, $a = \frac{\pi}{4} d^2 = 0.7854 \times \left( \frac{1.5}{1,000} \right)^2 = 1.77 \times 10^{-6}$ m²,

and $\theta_1 = (235 - 25) = 210°C$. 
Now, assuming the copper wires to be infinitely long, eqn. (1.51b) may be used for estimating heat input at the joint, where \( I = \alpha \). Thus, heat input,

\[
H = \sqrt{\frac{h_pK_a}{\theta_1}} (1 - e^{-\mu x})
\]

\[
= \sqrt{15 \times 0.0047 \times 390 \times 1.77 \times 10^{-6} \times 210 \times (1 - 0)} = 1.465 \text{ watts}
\]

**Problem - 13:** Heat generated in the bearing by friction causes the temperature at the end of the shaft 60 mm in diameter to rise 60°C above the ambient temperature. What is the temperature distribution along the shaft and what is the amount of heat transferred through it if heat transfer coefficient \( h \) is 6 W/m²°C, thermal conductivity of the material is 50 W/m°C and the shaft is assumed to be of infinite length?

For infinite rod, from eqn. (1.49), \( \theta = \theta_1 e^{-\mu x} \)

where, \( \mu = \sqrt{\frac{h_p}{Ka}} = \sqrt{\frac{6 \times \pi \times 0.06 \times 4}{50 \times \pi \times 0.06 \times 0.06}} = 2.8284 \text{ l/m} \)

Hence, the temperature distribution along the length of the shaft may be determined from, \( \theta = 60 \times e^{-2.8284 x} \) where, \( 0 < x < \infty \)

and heat flow from eqn. (1.51a), i.e.,

\[
H = Ka \mu \theta_1 (1 - e^{-\mu x})
\]

\[
= 50 \times \frac{\pi}{4} \times (0.06)^2 \times 2.8284 \times 60 \times (1 - e^{-2.8284 \times \infty})
\]

\[
= 24 \text{ watts.}
\]

**1.5.3 Heat transfer between two fluids separated by a metal wall:** Heat-exchange system usually consists of combination of the radiation, conduction and convection methods of heat transfer; of these, the fluid to fluid heat-exchange process through a separating wall, is perhaps the most common. In fig. 1-16, heat is transferred through a flat plate with hot fluid on one side and cold fluid on the other. Consider a steel plate 1 cm thick with thermal conductivity 37.2 watts m°C and \( T_1 \) and \( T_2 \) as 700°C and 150°C respectively. Assuming for the moment that the metal temperature is the same as that of fluid in contact with it,

\[
H = \frac{37.2 \times 1 \times (700 - 150)}{1 \times \frac{1}{100}} = 20,460,000 \text{ watts}
\]

In actual practice it is found to be 24,400 watts and hence actual temperature drop \( T_1' - T_2' \) is about 6.56°C. Thus, it is seen that the bulk of the temperature drop is in thin layers of the gases on either side. These thin layers are the main resistance to heat flow and this is generally accounted for by heat transfer coefficient. It is sometimes called film coefficient, \( h \). It may be observed that this film coefficient is equivalent to coefficient of conductivity of the fluid layer.

If \( h_1 = \) film coefficient for the hot fluid to metal (watts/m²°C),

\( h_2 = \) film coefficient for the metal to cold fluid (watts/m²°C), and

\( U = \) overall heat transfer coefficient between hot fluid and cold fluid, (watts/m²°C),
Hence, rate of heat flow per unit area,

\[ H = h_1 \left( T_1 - T_1' \right) \text{ for hot fluid film} \]

\[ = \frac{K(T_1' - T_2')}{S} \text{ for plate} \]

\[ H = h_2 \left( T_2' - T_2 \right) \text{ for cold fluid film} \]

\[ \therefore (T_1 - T_1') = \frac{H}{h_1}; (T_1' - T_2') = \frac{HS}{K}; (T_2' - T_2) = \frac{H}{h_2} \]

Adding above three relations and simplifying, we get,

\[ T_1 - T_2 = H \left( \frac{1}{h_1} + \frac{S}{K} + \frac{1}{h_2} \right) \text{ or} \]

\[ H = \frac{T_1 - T_2}{\frac{1}{h_1} + \frac{S}{K} + \frac{1}{h_2}} \]

It has been noted that the film coefficient \( h \) is equivalent to the conductance of the fluid layers adjacent to the surface of the solid wall. The corresponding resistance of these fluid layers is, therefore, \( 1/h \). The resistance of the fluid film may be added to the resistance of the wall to give the overall resistance to heat flow between the two fluid layers. Thus, from eqn. (1.52) it may be observed that

\[ \frac{1}{h_1} + \frac{S}{K} + \frac{1}{h_2} \text{ is the overall resistance.} \]

Thus, from the definition of overall heat transfer coefficient,

\[ H = U \left( T_1 - T_2 \right) \]

where, \( U \) is the overall conductance of the entire wall including the effect of the resistance offered by the fluid films, and is also called the overall heat transfer coefficient.

Comparing eqns. (1.52) and (1.53),

\[ \frac{1}{U} = \frac{1}{h_1} + \frac{S}{K} + \frac{1}{h_2} \]

or for composite walls

\[ \frac{1}{U} = \frac{1}{h_1} + \sum \frac{S}{K} + \frac{1}{h_2} \]

Owing to their low thermal conductivity, the surface coefficient operating in the case of gas flow are very much smaller than those for water and other liquids whose viscosities are not high. A gas flowing at, say, 5 m/sec, across a tube may have a surface coefficient of the order of 50 to 100 watts/m²°C, whilst that for water flowing through the tube at about 2 m/sec. is about 50,000 watts/m²°C.

For a cylindrical tube of inner radius \( r_1 \) and outer radius \( r_2 \), the overall coefficient may be expressed as

\[ \frac{1}{U r_m} = \frac{1}{h_1 r_1} + \frac{\log_e \left( \frac{r_2}{r_1} \right)}{K} + \frac{1}{h_2 r_2} \]

Here \( U \) is the overall heat transfer coefficient based on some mean radius \( r \), but since the real interest lies in the value of product \( U r_m \), the radius \( r_m \) need not be specified.
If the temperature drop across the pipe wall is neglected,
\[
\frac{1}{U_{rm}} = \frac{1}{h_1 r_1} + \frac{1}{h_2 r_2}
\]  
... (1.57)

If in addition, the thickness of pipe is negligible, i.e. \( r_2 = r_1 \),
\[
\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}
\]  
... (1.58)

Lastly, for a composite pipe having \( n \) layers, separating hot and cold fluids,
\[
\frac{1}{U_{rm}} = \frac{1}{h_1 r_1} + \sum_{n=1}^{n} \log_e \left( \frac{r_{n+1}}{r_n} \right) \frac{1}{K_n} + \frac{1}{h_2 r_{n+1}}
\]  
... (1.59)

Problem – 14: A pipe having 5 cm bore and 6.25 cm outer diameter is covered by two layers of lagging 11.25 cm and 14 cm diameter and having thermal conductivity of 0.164 and 0.0446 watts m°C respectively. Pipe material has thermal conductivity of 20 watts/m°C.

The pipe carries superheated carbon dioxide flowing at the rate of 80 kg/hr. If the heat transfer coefficient between surrounding and outer surface of lagging in 37 watts/m² °C, find the rise in temperature of CO₂ per metre run of pipe if temperature of surroundings is 17°C and average temperature of the carbon dioxide is –25°C. The heat transfer coefficient between metal and CO₂ may be taken as 50 watts/m°C. Specific heat of CO₂ is 0.9 KJ/kg °C.

Using eqn. (1.59), overall heat transfer coefficient for three layers ( \( n = 3 \) ) is given by,
\[
\frac{1}{U_{rm}} = \frac{1}{h_1 r_1} + \log_e \left( \frac{r_2}{r_1} \right) \frac{1}{K_1} + \log_e \left( \frac{r_3}{r_2} \right) \frac{1}{K_2} + \log_e \left( \frac{r_4}{r_3} \right) \frac{1}{K_3} + \frac{1}{h_2 r_4}
\]
\[
= \frac{10}{50 \times 2.5} + \frac{\log_e \left( \frac{3.125}{2.5} \right)}{20} + \frac{\log_e \left( \frac{5.625}{3.125} \right)}{0.0446} + \frac{\log_e \left( \frac{10}{5.625} \right)}{0.164} + \frac{100}{37 \times 7}
\]
\[
= 0.8 + 0.01115 + 13.15 + 1.336 + 0.386
\]
\[
= 15.68 \text{ m°C / watts}
\]

Now, heat flow rate from surrounding to CO₂ per metre run of pipe is given by
\[
H = -2\pi U_{rm} \frac{1}{T_2 - T_1} = -2\pi \times (U_{rm}) \times 1 \times (17 + 25)
\]
\[
= -2\pi \times \frac{1}{15.68} \times 1 \times 42 = -16.9 \text{ watts} = -16.9 \text{ J/s}
\]

Thus, temperature rise of CO₂ per metre length of the pipe
\[
= \frac{16.9 \times 3600}{80 \times 0.9 \times 1000} = 0.845 ^\circ\text{C}
\]

1.6 Heat Dissipation from Lagged Steam Pipes

The heat loss from the steam pipe surface to the atmosphere takes place by convection, conduction, and radiation. This loss is usually given in the form of an empirical coefficient in heat units, per unit time, per unit surface area, per degree difference of temperature
beetwen temperature of exposed surface and atmosphere. The coefficient is known as coefficient of emissivity. Thus, rate of heat loss can be, expressed as

\[ H = h_e (T_1 - T_a) \times A \quad ... (1.60) \]

where, \( h_e \) = coefficient of emissivity.

The overall heat transfer coefficient, \( h_o \) is merely the quantitative design characteristic of the complex process of heat transfer taking into consideration combined effect of all three basic modes of heat transfer.

1.6.1 Lagged Steam pipe: The object of lagging is to reduce the heat loss from the pipe surface by providing an insulating layer called lagging, through which heat has to flow. Fig. 1-18 shows the lagging round a steam pipe of inner radius \( r_1 \) and negligible thickness, at a temperature \( T_1 \), lagged up to a radius \( r_2 \) having a surface temperature \( T_2 \) which is exposed to environment of air at a temperature \( T_a \). Neglecting the temperature drop across the pipe wall, heat passing through the lagging will be given by equation (1.10) and this must be equal to the heat loss from the surface. If \( h_e \) is the emission coefficient (by radiation, convection and conduction) from the surface per unit area per degree difference in temperature in unit time, then, rate of heat conducted through the lagging material from eqn. (1.10),

\[ H = \frac{2\pi l K (T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \]

The same rate of heat flow is emitted from the external surface, thus

\[ H = h_e 2\pi r_2 l (T_2 - T_a) \]

Equating above two heat flows and re-arranging,

\[ T_1 - T_a = H \left\{ \frac{1}{h_e r_2} + \frac{\log_e \left( \frac{r_2}{r_1} \right)}{K} \right\} \]

\[ : H = \frac{2\pi l (T_1 - T_a)}{\left\{ \frac{1}{h_e r_2} + \frac{\log_e \left( \frac{r_2}{r_1} \right)}{K} \right\}} \quad ... (1.61) \]

It will be seen from fig. 1-18 that as \( r_2 \) increases, the areas of successive layers of lagging increases, so that the resistance is offered by each succeeding layer of thickness \( \delta r \). Moreover, the external area from which the heat is finally lost also increases.

Differentiating the denominator of equation (1.61) with respect to \( r_2 \) and equating to zero gives the relation \( r_2 = K / h_e \), and its substitution in the second differential gives
positive expression. This is the condition for a minimum value of the denominator, and a maximum value of the heat loss. For asbestos, the critical radius needs normally very small thickness – a fraction of centimetre only – especially when radiation losses are high. As \( r_2 \) is increased beyond this value (fig. 1 – 19), the heat losses diminish continuously, but the saving of heat must be considered with reference to the cost of the lagging.

Problem – 15 : A 10 cm outer diameter pipe carrying saturated steam at 17.6 bar is lagged with magnesia up to 20 cm diameter and further lagged with laminated asbestos up to 25 cm diameter. The whole pipe is further covered by a layer of canvas. If the temperature under the canvas is 20°C, find the mass of steam condensed per day of 24 hours per 100 metre length of pipe. Take coefficient of thermal conductivity of mangnesia and asbestos as 0.06 and 0.071 watts/m°C respectively.

Find the saving in coal per day per 100 m length of pipe due to this lagging if calorific value of coal is 32,659 KJ/kg. Boiler efficiency may be taken as 75%. Heat losses from the bare pipe are 14·7 watts/m² °C. Air Temperature is 20°C.

At 17·6 bar, \( t_s = 250 ^\circ\text{C} \), \( L = 1921·83 \text{ KJ/kg} \) (from steam tables by interpolation).

Using eqn. (1.12), \( H = \frac{2\pi l(T_1 - T_2)}{\log_e \left( \frac{r_2}{r_1} \right)} \sum \frac{1}{K} \)

Neglecting resistance of pipe,

\[
\sum \frac{1}{K} = \frac{\log_e \left( \frac{10}{5} \right)}{0·06} + \frac{\log_e \left( \frac{12·5}{10} \right)}{0·071} = 11·6 + 3·25 = 14·85
\]

\( \therefore \) Heat flow per 100 metre length of pipe

\[
= \frac{2\pi \times 100 (205 - 20)}{14·85}
\]

\( = 7,830 \text{ watts} = 7,830 \text{ Joules/sec.} \)

\( \therefore \) Heat loss per day = \( 7,830 \times 3600 \times 24 = 6·7651 \times 10^8 \text{ Joules/day} \)

\( \therefore \) Mass of steam condensed per day = \( \frac{6·7651 \times 10^8}{1921·83 \times 1000} \) = 352 kg/day

Now, heat loss from the bare pipe per day per 100 metre length of pipe is

\( = 14·7 \times \pi \times 0·1 \times 3600 \times 24 \times (205 - 20) = 7·38 \times 10^9 \text{ J/day} \)

\( \therefore \) Saving in heat due to lagging = \( 7·38 \times 10^9 - 6·7651 \times 10^8 \)

\( = 6·7 \times 10^9 \text{ J/day} \)

\( \therefore \) Saving in coal = \( \frac{6·7 \times 10^9}{32659 \times 1000 \times 0·75} \) = 273 kg/day
1.7 Heat Exchangers

Heat transferers or exchangers are necessary in many thermodynamic processes. The apparatus effecting these heat exchanges are termed heat exchangers. Here, the study of the basic principles which are must in the design of heat exchangers will be undertaken.

As mentioned earlier, heat exchanger is a very important part of the engineering plant. The cost of heat exchanger increases rapidly with the reduction in temperature difference between the fluids, while low value of temperature difference is desirable for minimising reversibility and hence inefficiency. Thus, designer must make a decision as to the economic limit to which compliance with thermodynamic principles may go.

There are widely varying conditions under which heat exchanges take place. This has led to development of many special and general types of heat exchangers. The temperature difference and pressure drop are the main considerations for design. However, corrosiveness, toxicity and scale-forming tendency, in addition to the thermal properties of the substances, must be considered while designing the exchanger. There are also economic considerations which include factors such as, initial cost, necessary space, life of the unit and ease of maintenance.

Heat exchangers may be classified as either recuperator type or regenerative. Recuperator types are natural heat exchanger units with continuous action, while in regenerative exchangers, the hot and cold fluids streams pass alternately over the heat exchanging surface. This may use a rotating cylindrical metal grid which is heated while passing through the hot fluid stream and which then gives up this heat to the cold fluid when passing through the cold fluid stream. These are used in power plants.

In order to visualize the effective mean temperature difference (not algebraic mean), the temperature distribution in various types of heat exchangers will be considered. Subscripts 1 and 2 refer to the inlet and outlet conditions of fluid respectively. Capital $T$ refers to hot fluids temperature while lower $t$ refers to cold fluid temperature. The difference in fluid temperatures at any stage is denoted by $\theta = (T - t)$.

All heat exchangers may be classified as follows:

1.7.1. Parallel flow Heat Exchanger: In this the fluids flow parallel to each other and in the same direction over the separating wall as shown in Fig. 1-20. Many devices such as water heaters, oil heaters fall in this class. It may be observed that the temperature of the hot fluid decreases, while that of cold fluid increases, along the length of the parallel flow heat exchanger, starting from hot end.

Total heat exchange over the entire length of the heat exchanger may be given by
HEAT TRANSFER

\[ H_L = U \cdot A \cdot \theta_m = U \times \pi D_m L \times \theta_m \] ... (1.62)

where, \( U \) = overall heat transfer coefficient,
\( D_m \) = log-mean diameter of the pipe separating the fluids, and
\( \theta_m \) = mean temperature difference to be defined later.

Here, initial temperature difference between the between fluids = \( \theta_1 = T_1 - t_1 \)
Final temperature difference between the fluids = \( \theta_2 = T_2 - t_2 \)

At a distance \( x \) from hot end of heat exchanger, temperature difference between the fluids = \( \theta_1 = T - t \)

1.7.2 Counter flow Heat Exchanger : In this case, the fluids flow in the opposite directions to one another. This possibly is the most favourable kind of fluid heater and cooler. A fluid flow and temperature distribution diagram is shown in fig. 1–21 (on page 38).

Here, initial temperature difference between the fluids = \( \theta_1 = T_1 - t_2 \), and
Final temperature difference between the fluids = \( \theta_2 = T_2 - t_1 \), while

Temperature difference between the fluids at a distance \( x \) from the hot end of heat exchanger = \( \theta = T - t \)

1.7.3 Evaporators and Condensers : In this case, one fluid remains at constant temperature while changing its state from liquid to vapour or from vapour to liquid while temperature of the other fluid changes as it flows along the length of the exchanger, the fluids flowing parallel to each other in any case. A fluid flow and temperature distribution diagram for a condenser in parallel flow arrangement is shown in fig. 1–22 (on page 38). It may be observed that even if direction of flow of vapour is reversed, the temperature diagram remains unchanged for both the hot and cold fluids. Thus, evaporators and condensers are insensitive to the direction of flow of the fluids.

Total heat exchange over the entire length of the exchanger is given by eqn. (1.62), here also.

Here, initial temperature difference between the fluids = \( \theta_1 = (T_s - T_1) \), final temperature difference between the fluids = \( \theta_2 = (T_s - T_2) \), while temperature difference between the fluids at a distance \( x \) from the left end = \( \theta = (T_s - t) \).

1.7.4 Cross flow Heat Exchangers : Another arrangement of a heat exchanger is to have the two fluids flowing at right angles to each other on either side of the separating surface, known as cross flow heat exchanger. Mathematical analysis of such an exchanger is rather complex since the fluid temperatures vary both in the direction of flow and also normal to it, if the fluids are not mixed in the direction normal to flow direction. Cross flow heat exchangers are popular particularly where space is limited, as for example, in mobile power plants like automobiles and aeroplanes.

1.7.5 Direct contact Heat Exchangers and Regenerators : There are the special kinds of exchangers designed for specific applications. Direct contact heat exchangers are usually of counter flow type where separating walls are absent. These exchangers are more effective and are used where mixing of the fluids is permissible. Evaporative coolers and cooling towers are the examples of this type, used extensively in refrigeration industry. Regenerators are the exchangers made up of grids of a material having high thermal storage capacity, through which, hot and cold fluids pass alternately, hot fluid heating the grid while flowing through it and cold fluid absorbing heat from the grid, in its turn, while flowing through the same grid. These exchangers are popular in turbojet power plant applications.
1.7.6 Logarithmic Mean Temperature Difference: For the proof of formula for effective mean temperature difference, consider a heat exchanger of length $L$ and logarithmic-mean diameter $D_m$ and take a point at a distance $x$ from the hot end of the heat exchanger.

Let $W_h$ = mass flow rate of hot fluid per second, $W_c$ = mass flow rate of cold fluid per second, and $U$ = overall heat transfer rate based on diameter $D_m$.

Then at a distance $x$ from the hot end,

Heat given by hot fluid for length $dx$ is, $dH = - W_h \times c_h \times dT$ \hfill (1.63)

where, $c_h$ = specific heat of hot fluid.

Heat received by the cold fluid for length $dx$ is $dH = W_c \times c_c \times dt$ \hfill (1.64)

where, $c_c$ = specific heat of cold fluid.

As $dt$ for parallel flow is positive and that for counter flow is negative, positive sign
in above formula is for parallel flow, while negative sign is for counter flow.

The heat flow rate, \(dH\) passes from hot fluid to cold fluid through the pipe wall.

\[\therefore dH = U \times \pi D_m \times dx \times \theta\] \hspace{1cm} \text{(1.65)}

Now, \(\theta = T - t\) \hspace{1cm} \therefore d\theta = dT - dt

Substituting values of \(dT\) and \(dt\) from equations (1.63) and (1.64),

\[d\theta = dH \left[ - \frac{1}{W_h c_h} \pm \frac{1}{W_c c_c} \right] = -dH \mu\] \hspace{1cm} \text{(1.66)}

In the above equation, \(\mu\) is the thermal capacity parameter defined by

\[
\mu = \frac{1}{W_h c_h} \pm \frac{1}{W_c c_c}
\]

Integrating eqn. (1.66), \(\theta_1 - \theta_2 = \mu H_L\)

where \(H_L\) = rate of heat transfer for full length of the exchanger

\[= U \times \pi D_m \times L \times \theta_m\]

where \(\theta_m\) = effective mean temperature difference

\[\therefore \theta_1 - \theta_2 = \mu U \times \pi D_m \times L \times \theta_m\] \hspace{1cm} \text{(1.67)}

Again from eqns. (1.65) and (1.66), \(d\theta = -\mu U \times \pi D_m \times dx \times \theta\)

i.e., \(\frac{d\theta}{\theta} = - \mu U \times \pi D_m \times dx\)

Integrating, \(\log_e \left(\frac{\theta_1}{\theta_2}\right) = \mu U \times \pi D_m \times L\) \hspace{1cm} \text{(1.68)}

Combining eqns. (1.67) and (1.68), we get, \(\theta_m = \frac{\theta_1 - \theta_2}{\log_e \left(\frac{\theta_1}{\theta_2}\right)}\) \hspace{1cm} \text{(1.69)}

It may be observed that the expression for the logarithmic mean temperature difference (LMTD) for counter flow or parallel flow exchangers is the same and involves only the temperature difference at the entrance and exit of the exchanger. However, it may be noted that for a given set of entrance and exit temperatures, the logarithmic mean temperature difference is greater in case of counter flow heat exchangers and therefore less area of heat transfer is needed. Further, in case of condensers and evaporators, both the directions of flow, i.e. parallel or counter flow, result in same temperature difference.

The third general arrangement of a heat exchange system is to have the two fluids flow at right angles to each other on either side of the separating surface known as cross-flow arrangement. The mathematical analysis of the mean temperature difference for cross-flow exchangers is rather complex, and is not presented here. The use of cross-flow exchangers is usually necessary when space is limited, as for example, in aircraft.

**Problem - 16**: A liquid to liquid counter flow exchanger is used to heat cold fluid from 50°C to 155°C. Assuming that the hot fluid enters at 260°C and leaves at 205°C, calculated the logarithmic-mean temperature difference for the heat exchanger.

Here, \(\theta_1 = (T_1 - t_2) = 260 - 155 = 105°C\) and
\[ \theta_2 = (T_1 - t_2) = 205 - 50 = 155^\circ \text{C} \]

Using eqn. (1.69), logarithmic mean temperature difference,
\[ \theta_m = \frac{\theta_1 - \theta_2}{\log_e \left( \frac{\theta_1}{\theta_2} \right)} = \frac{155 - 105}{\log_e \left( \frac{155}{105} \right)} = 129^\circ \text{C} \]

**Problem - 17:** A two-pass surface condenser is required to handle the exhaust from a Steam turbine developing 14,300 kW with a consumption rate of 14.66 kg/kWhr. and to maintain a steady vacuum of 73.6 cm Hg. (Barometer 76.2 cm Hg). Find, (a) condenser surface area required, (b) circulating water flow in litres per minute, (c) total number of tubes, and (d) length of tubes between tube plates if the following data is assumed:

Mean water velocity 3 m/sec.; water inlet temperature 15.8°C; difference between condensate temperature and water outlet temperature 2.8°C; condensate temperature 1.7°C below vacuum temperature; quality of exhaust steam 0.9; heat transmission coefficient 3.93 KJ/m²s°C based on outer tube surface; tube outside diameter 3.84 cm and tube thickness 0.44 cm.

(a) Steam consumption = \[ \frac{4.66 \times 14,300}{3,600} = 18.51 \text{ kg/sec.} \]

From steam tables, saturation temperature \( t_s \) corresponding to \( \frac{762 - 736}{750} = 0.035 \) bar is 26.5°C and \( L = 2,438.1 \text{ kJ/kg} \) (by interpolation).

.: Water outlet temperature, \( t_2 = 26.5 - 2.8 = 23.7^\circ \text{C} \).

Total heat to be removed from steam, \( H = 18.51 \times (2438.1 \times 0.9 + 1.7 \times 4.187) = 45,157.5 \text{ KJ/sec.} \)

Now, \( T_1 = 26.5 + 1.7 = 28.2^\circ \text{C}, \ T_2 = 26.5^\circ \text{C}, \ t_1 = 15.8^\circ \text{C}, \)
\( t_2 = 23.7^\circ \text{C} \) and \( \theta_1 = (T_1 - t_2) = 28.2^\circ \text{C} - 23.7^\circ \text{C} = 4.5^\circ \text{C}, \)
\( \theta_2 = (T_2 - t_1) = 26.5^\circ \text{C} - 15.8^\circ \text{C} = 10.7^\circ \text{C}, \) and \( t_2 - t_1 = 23.7^\circ \text{C} - 15.8^\circ \text{C} = 7.9^\circ \text{C}. \)

Using eqn. (1.69), Logarithmic mean temperature difference (LMTD),
\[ \theta_m = \frac{\theta_1 - \theta_2}{\log_e \left( \frac{\theta_1}{\theta_2} \right)} = \frac{4.5 - 10.7}{\log_e \left( \frac{4.5}{10.7} \right)} = 7.16^\circ \text{C} \]

Thus, condenser surface area required \( A = \frac{H}{U \theta_m} = \frac{45157.5}{3.93 \times 7.16} = 1,605 \text{ m}^2 \)

(b) Circulating water flow, \( W = \frac{H}{K \times (t_2 - t_1)} \)
\[ = \frac{45157.5 \times 60}{4.187 \times 7.9} = 81,913 \text{ litres/min.} \]

(c) Now, considering mass flow of water, number of tubes required per pass
\[ W = \frac{W}{w \times a_v} = \frac{81,913}{1,000 \times \frac{\pi}{4} \left[ 0.0384 - (0.0044 \times 2) \right]^2 \times 3 \times 60} = 661 \]

:. Total number of tubes required = \( 2 \times 661 = 1,322 \)

(d) Now, as surface area required \( A = n \times \pi D_0 l \)
where, \( n \) = no. of tubes, \( D_0 \) = outside diameter of tube and \( l \) = length of each tube, we get,

\[
I = \frac{A}{n \pi D_0} = \frac{1,605}{1,322 \times \pi \times 0.0384} = 10.07 \text{ metres}
\]

**Tutorial – 1**

1. Fill in the gaps to complete the following statements:
   (a) Conduction in ______ other than metals due to longitudinal oscillation, in ______ due to diffusion of free electrons and in ______ due to elastic impacts of molecules.
   (b) ______ appears to be the best conductor among gases; helium has slightly higher value.
   (c) If the entire incident radiant energy passes through the body, such bodies are called _____ or ______.
   (d) Wien’s law relates \( T \) and \( \lambda_m \) by the expression
   \[
   T\lambda_m = \frac{1}{\text{[mm} - \text{K]}}.
   \]
   (a) Solids, metals, gases, (b) Hydrogen, (c) absolutely transparent, diathermanous, (d) 2.898

2. Choose correct phrases to complete the following statements:
   (a) Logarithmic-mean temperature difference for a parallel flow heat exchanger in comparison with a counter flow heat exchanger is (i) greater, (ii) less, (iii) same.
   (b) Emissivity of a black body in comparison with a polished non-black body (i) is higher, (ii) is same, (iii) is lower, (iv) varies from body to body.
   (c) An increase in the temperature of metals, results in (i) corresponding decrease in thermal conductivity, (ii) corresponding increase in thermal conductivity, (iii) no change in thermal conductivity.
   (d) According to Kirchhoff’s law, (i) Ratio of emissive power to absorptivity, for all bodies is same and is equal to the emissive power of a black body, (ii) Emissive power does not depend upon temperature, (iii) Emissive power and absorptivity are constant, (iv) Emissive power depends upon square of absolute temperature.
   (e) A gray body is one (i) whose absorptivity varies with temperature, (ii) whose absorptivity and emmissivity varies with wave length, (iii) whose absorptivity does not vary with temperature and wave length of the incident ray, (iv) whose absorptivity varies with temperature and wave length of the incident ray.
   (f) For insulated pipe at critical radius (i) The rate of heat transfer by conduction equals the rate of heat transfer by convection at the surface and is minimum, (ii) It can be subjected to maximum pressure, (iii) The heat flow rate is minimum, (iv) The rate of heat transfer by conduction equals the rate of heat transfer by convection at the surface and is maximum.
   (g) The negative sign in the heat flow equation by conduction,
\[ H = -kA \frac{dT}{dS} \] indicates that

(i) Heat transfer can occur in the reverse direction, (ii) For extremely poor conductors like gases, the transfer of heat is staggered, (iii) The temperature decreases with increasing thickness of material, (iv) The temperature of any body decreases due to conduction, convection, and radiation of heat from the surface of the source.

(h) For a black body
(i) the absorptivity is one, (ii) the reflectivity is one, (iii) the transmissivity is one, (iv) all of the above.

3. (a) Explain by giving illustrations that in practice transfer of heat is the combined effect of three basic modes of heat transfer, viz. conduction, convection and radiation.
(b) Discuss the importance of heat transfer in various fields of engineering.

4. (a) Explain the mechanism of heat transfer by conduction in solids and fluids. "Why is steel better conductor of heat than insulating bricks?" — Discuss.
(b) The inner and outer surfaces of a furnace wall 30 cm thick made of refractory bricks \((K = 1.35 \text{ watts/m°C})\), are at 1,650°C and 320°C respectively. Find the reduction in heat loss through the wall to be obtained by adding 30 cm thickness of insulating bricks for which \(K\) is 0.3 watts/m°C, assuming the inside surface temperature of refractory bricks to remain fixed at 1,650°C. The temperature of outer surface of the bricks may be taken as 27°C.

(b) A steam pipe line, 11.5 cm outside diameter, is covered with two layers of different
materials. The first layer is 5 cm thick having thermal conductivity of 0.053 Joules/metre sec °C. The second layer is 3 cm thick and has a conductivity of 0.75 Joules/m sec °C.

Outside surface temperature of steam pipe is 235°C and that of the outer surface of lagging is 38°C. Calculate the heat loss per metre length of pipe per second, and the temperature between the two layers of insulation. [102 Joules/sec; 43°C]

8. A sphere of radius 40 cm is lagged to a radius of 50 cm, inner and outer surface temperatures of the lagging being 230°C and 65°C respectively. Find the rate of heat leakage, if coefficient of thermal conductivity is 6.4 watts per m² per °C difference per cm thickness. [265.5 watts]

9. The thermal conductivity of a certain material varies with temperature \( T \) according to the following relation:

\[
K = a + bT + cT^3
\]

where \( a, b \) and \( c \) are constants.

Derive the expression for computing the heat loss per linear metre for a hollow cylinder made of this material. Assume that the inner and outer radii are \( r_1 \) and \( r_2 \) and temperatures \( T_1 \) and \( T_2 \) respectively and the cylinder ends are perfectly insulated.

\[
\frac{2\pi \left[a(T_1 - T_2) + \frac{b}{2}(T_1^2 - T_2^2) + \frac{c}{4}(T_1^4 - T_2^4)\right]}{\log_e \left(\frac{r_2}{r_1}\right)}
\]

10. If the thermal conductivity of ice is 2.09 J/m sec °C and density at 0°C is 910 kg/m³, find the time in which the layer of ice 3 cm thick on the surface of a pond will increase in thickness by 1 mm, when the temperature of the surrounding air is 20°C below zero. Take latent enthalpy of ice as 335 kJ/kg. [222.43 sec.]

11. (a) Explain the mechanism of heat transfer by radiation.
   (b) By qualitative reasons, establish the expression

\[
\alpha + \rho + \gamma = 1
\]

where \( \alpha = \text{absorptivity}, \ \rho = \text{reflectivity}, \ \text{and} \ \gamma = \text{transmissivity of a body.}

12. (a) Discuss the concept of "black body" in heat transfer.
   (b) State the Plank's law of emission of radiant heat. Explain its significance.

13. State Stefan-Boltzmann's law of total radiation from a black body. Discuss how this can be modified to take into account radiation from a non-black body.

14. Write short notes on the following terms as applied in heat transfer:
   (a) Black body, (b) Gray body, and (c) Non-black body.

15. (a) Define the following terms as applied to heat transfer by radiation:
   (a) Emmissivity, (b) Absorptivity, and (c) Reflectivity.
   (b) Estimate the net radiant interchange between unit areas of two parallel perfectly black planes, infinite in extent and at temperatures of 430°C and 650°C respectively.
16. Explain the term emissive power and absorptive power of a body. Deduce that at any temperature, the ratio of emissive power to the absorptive power of a substance is constant and is equal to the emissive power of perfectly black body.

17. (a) Differentiate critically between mechanisms of heat transfer by free convection and that of forced convection.

(b) Explain the principle of dimensional analysis. Establish by dimensional analysis, the expression for Nusselt number in terms of Grashof and Prandtl numbers for free convection.

18. Estimate the heat loss by natural convection from a horizontal heated plate 45 cm x 20 cm at 95°C to still air at 15°C. The film coefficient for free convection, \( h \) is given by

\[
h = 1.9655 (\theta)^{1/4} \text{ watt/m}^2 \cdot \text{°C}
\]

where \( \theta \) is temperature drop in the air film in °C.

19. The rate of heat loss from a hot cylindrical surface freely exposed to still air is given by

\[
h = 1.17 \left( \frac{\theta}{d} \right)^{0.25} \text{ watt/m}^2 \cdot \text{°C}
\]

where, \( \theta \) = temperature drop in air film, °C, and

\( d \) = diameter of the cylinder, metre.

Find the heat lost per metre length per hour for a bare pipe 22.5 cm diameter, carrying gas at 320°C, when the temperature of the atmosphere is 20°C. If the pipe is covered with a layer of material 7.5 cm thick and for which thermal conductivity is 0.089 watts/m °C, show that the outer surface temperature of the material will be 75.5°C and hence find the heat lost per metre of pipe length per hour. Neglect the temperature drop through the metal.

20. (a) Explain in detail the mechanism of forced convection. State different empirical relations established by different workers for forced convection.

(b) Estimate the average film coefficient of heat transfer at the water side of single pass steam condenser. The tubes are of 2.3 cm inside diameter and cooling water enters at 18°C and leaves at 19.4°C. The average water velocity is 2.15 metre/sec. The properties of water at mean temperature of 18.7°C may be taken as:

- Density = 1,000 kg/m³
- Specific heat \( c' \) = 4.187 kJ/kg°C
- Viscosity = 3.74 kg/m hr
- Thermal conductivity = 0.587 J/m s°C

The general relation for forced convection with usual dimensionless group is

\[
\{N_{tu}\} = 0.23 \{N_{re}\}^{0.8} \{N_{pr}\}^{0.4}
\]

21. (a) Establish the expression for temperature distribution along a bar of uniform-section and of finite length heated at one end and exposed to surrounding which is maintained at constant temperature.

(b) Two vessels of large thermal capacity, the first at a temperature of 125°C, and the second at a temperature of 50°C, are perfectly insulated and connected by
a non-insulated tie-bar, 2.5 cm diameter and 30 cm long. Find the heat flow rate from or to the second vessel, if the tie-bar is of (a) copper whose thermal conductivity is 335 watt/m°C, and (b) steel whose thermal conductivity is 40 watt/m°C. Assume that the atmospheric temperature is 20°C and the surface coefficient of heat transfer is 10 watt/m² °C in each case. Sketch the temperature distribution along the length of the tie-bar.

(a) 15.5 watt; (b) 1.27 watt

22. A 1.5 mm diameter wire is maintained at 150°C. Temperature of surrounding is 15°C. Show the effect of covering the wire with glass wool of conductivity 0.03 watt/m °C by plotting heat loss against thickness of covering. What is the critical radius of the covering? Take the coefficient of heat loss from the surface of the covering as 10 watt/m² °C and that for the bare wire as 15 watt/m² °C.

2.25 mm

23. Following particulars refer to heat transfer through boiler tubes:
   Temperature of hot gases, 850°C; temperature of water in the tubes, 180°C; thicknesses of boiler tubes, water film and gas film, 1.25 cm, 0.025 cm and 0.075 cm respectively; coefficients of conductivities of boiler tubes, water film and gas film, 48, 0.48, and 0.33 J/m sec °C respectively.

   Estimate the amount of heat transmitted per sq. metre, per minute.

   3,670 J

24. A steam pipe of 15 cm outer diameter is carrying saturated steam at 14 bar. The pipe is lagged with 5 cm thickness of material of thermal conductivity 0.1786 W/m°C. The exposed surface is found to be at a temperature of 76°C. Neglecting the temperature drop through the pipe wall, estimate the mass of steam condensed per metre length of pipe, per hour.

   0.544 kg

25. Calculate the thickness of insulating material of conductivity 0.3768 kJ/m hr°C necessary to reduce the heat loss from a hot water tank by 25%. Assume the coefficient of surface heat transfer by radiation and convection to be 6 kJ/m² hr °C for the exposed surface of the lagging and 31.4 kJ/m² hr °C for the unlagged tank. Assume surface area of unlagged tank = surface area of lagged tank = surface area for conduction.

   3.3 cm

26. An electric cable is insulated with the insulating material having thermal conductivity of 0.628 kJ/m hr K. The outer radius of the bare cable is 6 mm and that of the insulated cable is 8 mm. Find whether the insulation helps in cooling of cable and also find the value of the outer radius which would maintain the same temperature at the wire when there is no insulation. Assume the rate of heat flow from the surface to air as 41 kJ/m² hr K.

   [ Insulation helps in cooling wire as \( K > r_2 \); outer radius = 6 cm ]

27. (a) Trace the importance of heat exchangers for industrial use.

   (b) Classify various heat exchangers and draw diagrams showing temperature distribution along their lengths.

28. (a) Establish the expression for logarithmic mean temperature difference for heat exchanger.
(b) Steam enters a condenser at 0.14 bar, 0.94 dry and condensate leaves at the saturation temperature corresponding to this pressure. The flow rate of steam is 7,250 kg/hr and that of water is 1,22,500 kg/hr. If the water inlet temperature is 10°C, calculate logarithmic mean temperature difference. If the total cooling surfaces is 140 sq.m., calculate also the overall rate of heat transfer.

$$\text{[20.2°C; 5715 kJ/m}^2\text{hr °C]}$$

29. The tubes in a smoke tube boiler are 2.5 m long, 7.5 cm diameter, the mass flow of the gases through the tubes is 14.7 kg/s/sq.m and outlet gas temperature is 250°C. The water outside the tubes is at a temperature of 160°C. Determine, from first principles, the temperature at inlet to the tubes and check the result by direct employment of mean temperature difference. Take $k_p$ for gas as 1 kJ/kg°C and overall heat transfer coefficient, $U$ as 0.0716 kJ/m$^2$s °C.

$$\text{[327°C]}$$

30. In an ammonia-brine cooler, the brine having a specific heat of 0.68 is cooled from a temperature of 4.5°C to −4°C at the rate of 13,620 kg/hr. If the evaporation of the ammonia takes place at −8°C, determine the cooler surface required. Take the overall heat transfer coefficient as 4,187 kJ/m$^2$hr°C.

$$\text{[10.59 m}^2\text{]}$$

31. In a tubular heat exchanger, 1,650 kg of liquid is to be cooled per hour from a temperature of 77°C to 50°C, the specific heat of the liquid being 2.512 kJ/kg°C. The water supply temperature is 21°C and the final temperature is not to exceed 38°C. The overall co-efficient of heat transfer may be assumed to be 4,606 kJ/m$^2$ hr°C. Estimate the surface area of the tubes required when using (a) parallel flow, and (b) counter flow configuration.

$$\text{[(a) 0.85 m}^2\text{; (b) 0.73 m}^2\text{]}$$

32. Find the surface area required in a surface condenser dealing with 27,000 kg of steam per hour, 0.92 dry and at 0.042 bar pressure. Temperature of condensed water leaving the condenser is 29°C. The cooling water is heated from 13°C to 24°C in passing through the condenser. Assume that mean coefficient of heat transmission is 12,561 kJ/m$^2$ hr°C.

If this condenser is to have two water passes, composed of tubes 1.9 cm outer diameter and 0.122 cm thick, determine the length and number of tubes per pass. The water speed is to be limited to 1.5 m/sec.

$$\text{[508.62 m}^2\text{; 3.78 m length; 1,128 tubes per pass]}$$

33. A tubular counter-flow heat exchanger (economiser) is required to raise the temperature of 900 kg of water per hour from 32°C to 82°C. Hot flue gases are available at 315°C and at the rate of 1,110 kg/hr. Assume film coefficient, $h = 30$ W/m$^2$°C for gas to metal and 5,390 W/m$^2$°C for metal to water.

Calculate, neglecting diameter effects, (a) the length of 1.9 cm inside diameter tube required and (b) the number of tubes required, if the water velocity in the tubes is to be limited to 1 m/min. Take $k_p$ for gases as 0.9 kJ/kg°C.

$$\text{[(a) 2.72 m ; (b) 53]}$$